SCIENTIFIC AMERICAN
No. SUPPLEMENT 568

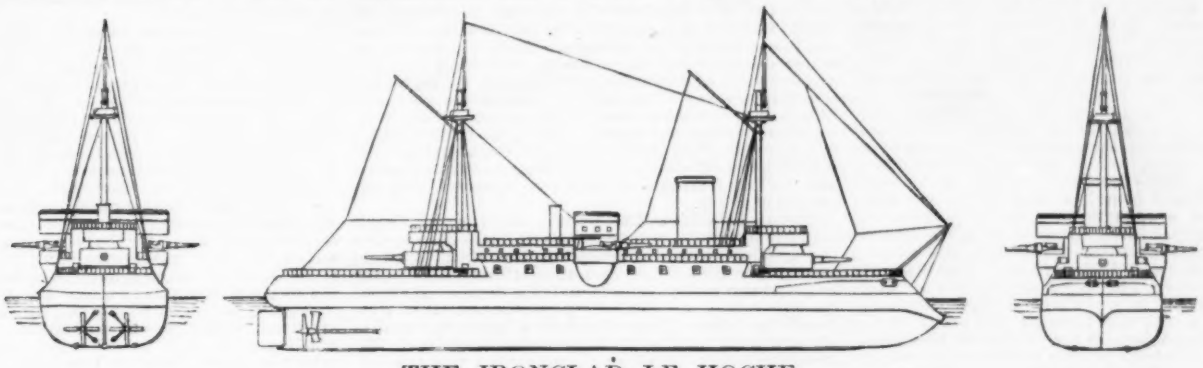
Scientific American Supplement, Vol. XXII., No. 568.
Scientific American, established 1845.

NEW YORK, NOVEMBER 20, 1886.

Scientific American Supplement, $5 a year.
Scientific American and Supplement, $7 a year.

LAUNCH OF THE FRENCH IRONCLAD LE HOCHE.

THE French ironclad Le Hoche was built after plans of Mr. Huin, one of the most distinguished engineers of the navy. She is 328 feet in length at the load water line, draws 26 feet of water, and has a displacement of 10,600 tons. Her defensive power is in keeping with her formidable armament, through the thickness of her armor plate, through her ironclad deck, through her ironclad turrets and ammunition passages, through her coffer dams filled with cellulose, her numerous water-tight compartments, her cellular structure, and her double bottom. She was built in one of the slips of the Caudan shipyard, on the east bank of the Scorf, and was launched on the 29th of September. Her engines and boilers are being constructed at Indret. The boilers are to be of the locomotive type, and the engines are being made after plans by Mr. Huin.



THE IRONCLAD LE HOCHE.

The method of launching selected by Mr. Huin was the same as that adopted for the Indomptable and the Formidable, *i. e.*, through the intermedium of the false keel. To the lateral keels were screwed stringpieces of oak, between which and the slides there was allowed a play of half an inch. As these pieces had to bear upon the slides in case the ship inclined to one side or the other, they, as well as the slides, were tallowed. The shoe beneath the false keel was formed of three courses of oak, 12 inches in width, giving a total width of 36 inches, upon which bore the entire weight of the ship. This shoe was divided into sections of such a length that the blocks could be put in place by merely removing two consecutive stocks. Between the false keel and the shoe were interposed billets of oak, which were set very close together and nailed to the shoe blocks. A large number of wedges, driven in with force, rendered the contact between the shoe and keel very perfect.

As the false keel was quite narrow, and as it was through this alone that the vessel bore upon her cradle, Mr. Huin made an endeavor to offset this by substituting for the oak billets, at intervals of 8 feet, oak timbers that passed through to the false keel, and the projecting ends of which supported a piece of oak which bore against the keel and thus increased the carrying surface.

For preventing the vessel from starting too soon after the removal of the shores, a system of wedges and keys in pairs was employed, and as this constituted the innovation at Lorient, it is well to dwell upon it a little. The launching way over which the shoe was to slide, and which rested directly upon the cross balks of the slip, was not continuous, but, over a length determined by calculation, was interrupted here and there in order to make room for the wedges. The friction, then, oak against oak, was considerable; in the last launches it was 0·32. When the tallow is of good quality, and is applied on the day of the launching, the friction in the greased part is but 0·06. The inclination of the slip is 1 to 12.

Were there no wedges, the force that would carry the ship along would be given by the equation

$$F = P (\sin \alpha - f \cos \alpha) = P \cos \alpha (\text{tg } \alpha - f)$$

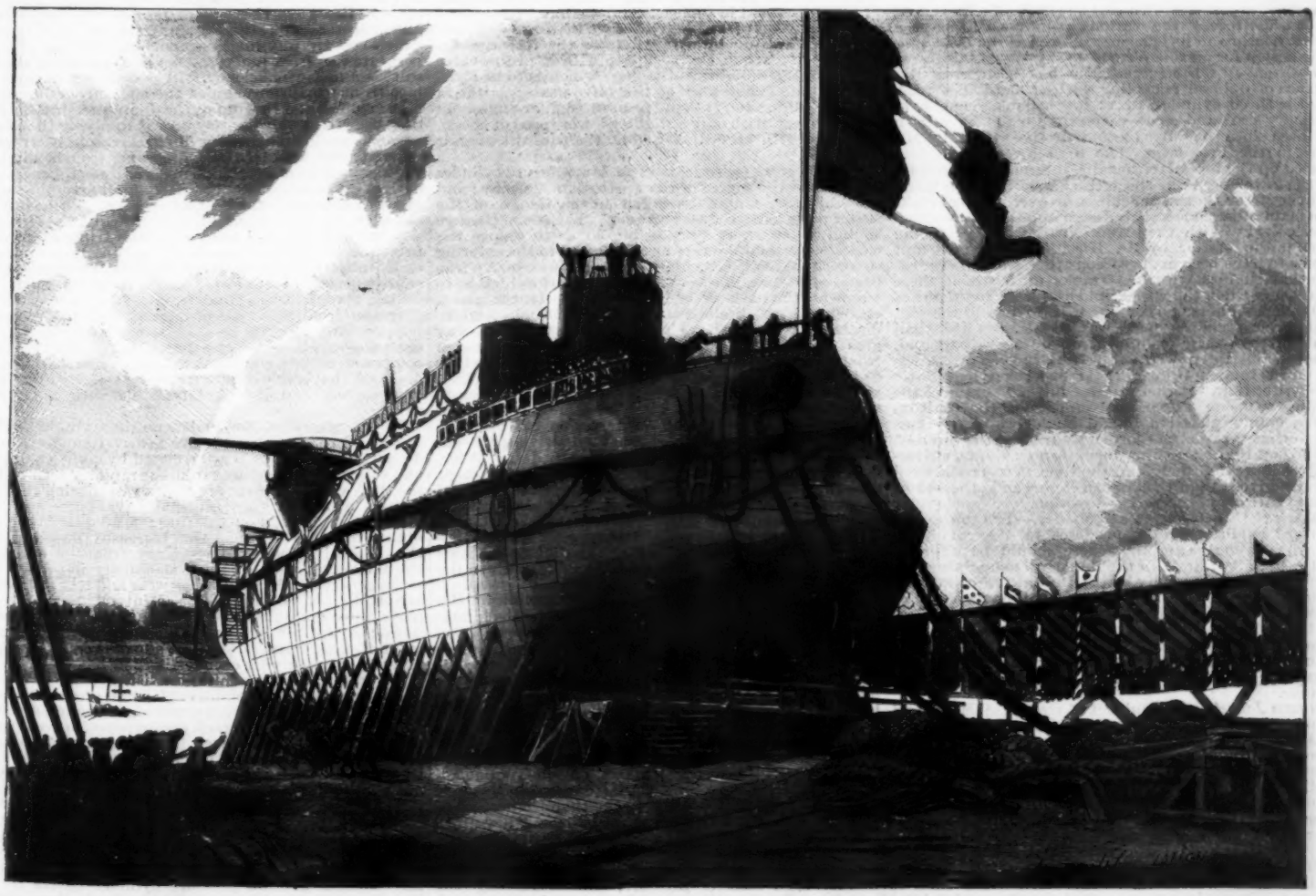
in which P = 4,300 tons, tg α = 0·083, and where we make f = 0·02, for the sake of precaution, instead of 0·06.

We then have F = 271 tons.

In this formula, as in the following ones, we shall take cos α = 1, seeing the small size of the angle α; we thus neglect only the square of the arc α.

From the equation

$$F = P (\text{tg } \alpha - f) \text{ or } \frac{F}{P} = \text{tg } \alpha = \text{tg } \alpha - f,$$



LAUNCH OF THE FRENCH WAR STEAMER LE HOCHE.

it results that if f were equal to tg α or to 0·0833, the ship would not budge. We shall thus succeed in determining the number of wedges necessary to prevent the vessel from starting.

Let x be the fraction of the length of the ship that they occupy; $1 - x$ will be the fraction occupied by the tallowed part, and we shall have

$$0{\cdot}0833 = (1 - x) \times 0{\cdot}02 + x \times 0{\cdot}33$$

or

$$0{\cdot}0633 = x \times (0{\cdot}32 - 0{\cdot}02) = 0{\cdot}3\,x.$$

$$x = \frac{0{\cdot}0633}{0{\cdot}3} = 0{\cdot}21.$$

As this result 0·21 is independent of the weight of the ship, it follows that for the inclination of 1 to 12 the wedges will prevent a vessel from starting if they occupy 0·21 of the length of any ship whatever.

For the Le Hoche, 0·27 was taken, and the result was certainly successful. In order to give the ship a start, she had to be raised in this part so as to suppress the friction or to reduce it to a minimum. To this effect there was arranged on each side of the wedges three strong wooden keys that were to be driven in by pigs of iron, weighing 110 pounds, supported by transverse slides, and each maneuvered simultaneously by six men.

Four o'clock in the afternoon was the hour set for the launch. At three o'clock, when but a single row of shores were standing, the ceremony of benediction was proceeded with. Then, one by one, the remaining shores were removed. The depth of water at the entrance to the slip being then 15½ feet, Mr Huin judged that that was sufficient, and that it was unnecessary to wait any longer, so he had the two dogs knocked away. After this the ship was held back only by the wedges, and as she did not budge, it was evident that this mode of fixation was efficient. In order to set the vessel in motion, order was given to drive in the smaller keys. The 288 men, each one at his post, gave, altogether and in unison, five blows, and then, after a delay of a moment, in order to judge of the effect, all these men were transferred to the larger keys. After three blows on these with the rams, a grating noise was heard, the great mass trembled, and the men had scarcely time to get out of the way when the ironclad began to move, with a quick motion at first, and then a slow one, through the effect of the unwinding of the cables and the parting of the breaking bosses. She had not yet left the slip when a thick cloud of smoke hid her for a few instants. When this cleared away she was seen afloat and immovable, having run to a distance of 150 yards. Everything passed off according to the engineer's expectations. An examination made after the launch showed that the tallow on the slides of the lateral keels was perfectly intact, thus indicating that the ship had borne only upon her false keel, and had listed neither to the right nor left.—*Abstract from Le Genie Civil.*

Our large engraving is from *Le Monde Illustre.*

TRIPLE EXPANSION MARINE ENGINES.

AT the recent Leeds meeting of the Institution of Mechanical Engineers, the following paper by the late Mr. Robert Wyllie, of Hartlepool, was read:

The last few years may be regarded as a transition period in the history of marine engineering, as the high pressure tripie expansion engine has now proved the successful rival of the double expansion compound. The object of the present paper is to bring forward the results of experience with this new type of engine, and to consider briefly the various points which have a direct bearing on its efficiency, as well as the most suitable design for marine purposes.

Position of Cylinders.—There has been great difference of opinion regarding the best method of placing the three cylinders in a triple expansion engine of ordinary size; and some very crude plans have been adopted since the introduction of the system by Mr. Kirk in the Propontis. The high pressure cylinder placed on the top of either the intermediate pressure or the low pressure has been tried, together with many schemes to lessen the difficulties of overhauling; while the main objections to this design appear to have been overlooked. With the exception of what was being done in the matter by Mr. Kirk, no further attempt seems to have been made to construct small power expansion engines on three cranks until about three years ago, when the writer undertook to build one of 700 indicated horse power which should fulfil the condition that no more space was to be occupied than would have been taken up with an ordinary compound. Engines of small power had previously been constructed on the tandem principle; but experience had shown the writer that, in order to take full advantage of the triple expansion system, an engine must be built on three cranks placed at equal angles; and the best proof that this was the correct solution of the problem is the fact that the arrangement is now almost universally followed, although, at the time, it was considered by the advocates of the tandem engine and others to be a step entirely in the wrong direction.

General Conditions of Efficiency.—The most important conditions to be considered in order to obtain an efficient engine are that there should be approximate equality: first, in the range of temperature in each cylinder; secondly, in the initial stress on each crank; and thirdly, in the indicated horse power of each engine. What may be termed the complements to these three essentials are: 1, steam jacketed cylinders; 2, cylinder ratios; 3, velocities of initial and exhaust steam; 4, clearance and compression; 5, receiver capacity; 6, piston speed; 7, order of sequence of cranks.

Steam Jackets.—The subject of steam jacketing, and of fitting cylinders with working barrels cast separately, has undergone considerable discussion among engineers, although the theoretical advantages of the system have long since been admitted. Keen competition seems evidently to have been one of the reasons why this fitting in many instances has been discontinued, in order to meet the demand for a cheap engine. Thorough investigation of the subject is beyond the scope of this paper; but the action of the cylinder surface, producing as it does an influence on coal consumption, deserves more attention than it often receives. In a single cylinder engine with a high ratio of expansion and a large range of temperature, it is well known that in each stroke there is excessive initial condensation, followed by partial re-evaporation. In triple expansion engines, where attention is paid to the equal division of the total range of temperature among the cylinders in which the successive stages of expansion take place, the benefits arising from the use of jackets are naturally not so great as in a single cylinder engine with a high ratio of expansion; but however carefully the triple engine may be designed, the jacketing of at least the intermediate and low pressure cylinders is essential to maximum efficiency. Beyond the actual economy resulting from steam jacketing, there are numerous practical advantages of a working barrel cast separately. The cylinder barrel being one of the most vital parts of an engine, it is essential that it should be of the finest material and of uniform quality, in order to prevent it from wearing unevenly, as an oval or uneven cylinder means a leaky piston and increased coal consumption. In the case of a cracked cylinder without an independent liner, the expense of a renewal is very great, while a new liner costs but little; and the additional facility given for heating up cylinders fitted with independent liners is a safeguard against accidents of that description.

Cylinder Ratios.—The ratio of cylinder capacities in a triple expansion engine depends on the pressure of steam and the type of engine. In cargo steamers, where economy of fuel is of vital importance and a large range of reserve power is not necessary, the high pressure cylinder should be of such a diameter that with a cut off of from 50 to 60 per cent. the theoretical absolute terminal pressure in the low pressure cylinder shall not exceed 10 lb. per square inch. In war ships, where a large range of power is sometimes required and economy of fuel is not so important, the high pressure cylinder should be larger in proportion, so that a higher mean pressure can be obtained. In triple expansion engines on three cranks, the intermediate cylinder should be so proportioned that, with 55 to 65 per cent. cut off, the powers, the ranges of temperature, and the initial stresses in the three cylinders may approach equality.

Steam Velocities.—To obtain even approximate equality in powers, temperatures, and stresses requires the greatest care in designing the steam passages throughout the engine; and unless the velocities of the steam at the various points and the degrees of cut off by the valves are carefully proportioned, it will be found that these three elements of economy and of efficient working are very far from being realized. The greater number of published results show that the subject is still inadequately appreciated. An expanded high pressure diagram taken from an engine with 135 lb. boiler pressure shows that the velocity in the passages was by no means high, but the pipes were indirect, and owing to specified requirements there were several altogether unnecessary valves; the result was a fall of pressure to 120 lb., or a loss of 15 lb. available initial pressure. When the objectionable bends were removed, the initial pressure in the diagram rose to 130 lb., thus raising the ratio of expansion and increasing the general efficiency. The stroke of the engine was 3 ft., and the revolutions 104 per minute. A high pressure expanded diagram from a cylinder of the same diameter and stroke shows the revolutions to be 70 per minute. This shows how small the initial drop may be, when the sources of loss are thoroughly appreciated.

Piston Valves.—The use of piston valves on the intermediate and low pressure cylinders, especially in engines of moderate power, serves to illustrate how the indirectness of the valve passages impairs the efficiency of the steam, and in some cases more than balances the beneficial effect of reduced friction in the machinery. An expanded diagram taken from the intermediate cylinder of a triple engine fitted with a piston valve shows considerable wire drawing, which is to be attributed to the indirectness of the valve passages; for in another diragram, taken from a similar engine in every respect, with only a slightly lower speed of steam, but fitted with an ordinary slide, the wire drawing is much less.

Low Pressure Cylinders.—The low pressure cylinder is of course the source of the greatest inefficiency; and too much care cannot be taken in the design of the steam ports and exhaust passages. The steam passage should be as short as possible, so as to reduce the clearance to a minimum; and the speed of the entering steam should not be so high as to cause excessive frictional resistance, nor the speed of exhaust so high as to augment the back pressure; consequently, the greatest efficiency is obtained when the revolutions and indicated horse power are not required to vary to any great extent. As an illustration, a diagram was taken from a low pressure cylinder which was designed for 60 to 65 revolutions per minute; on the lightship trial the engines were run at 78 revolutions, and the vacuum was then 1¾ lb. less than with the same cut off at 68 revolutions, the loss being entirely due to the excessive exhaust velocity consequent on driving the engines at a greater speed than they were designed for. Contracted or indirect exhaust passages in the high pressure and intermediate cylinders have the effect of causing a larger difference between the back pressure on one piston and the initial pressure on the next, thus diminishing the efficiency of the steam. An illustration of this defect is furnished by the diagrams which were taken from a triple expansion engine built in the North of England.

Cut Off.—The cut off necessary to maintain the equality of the three essentials for an efficient engine is governed to a great extent by the speed of the entering steam and the nature of the passages, inasmuch as the same amount of steam per stroke may enter the cylinder at the ordinary working number of revolutions per minute as at a higher speed with later cut off and larger port opening; and in the latter case the difference will be greater between the actual and the effective cut off. An expanded high pressure diagram was taken on a lightship trial, and shows the effect produced by a high velocity of steam on the volume admitted at each stroke, the actual cut off in this case being 8 per cent. later in the stroke than the virtual or effective cut off. In the intermediate and low pressure cylinders, too high a velocity of the entering steam will produce excessive frictional resistance, causing a drop in the expansion curve as well as an unduly high receiver pressure, thus disturbing the equality of temperatures and of initial stresses. A diagram was taken from the intermediate cylinder on a lightship trial with a high steam velocity through the ports, and another diagram from the same cylinder in the loaded ship; the cut off was equal in both instances, yet in the latter case the receiver pressure is 7 lb. lower, and the drop in the expansion curve is very considerably reduced. A three crank engine, with its more uniform twisting moment on the shaft, and its approximately equal initial loads, presents every facility for increased revolutions or higher piston speed; and as lighter machinery for the same power developed admits of additional capacity for carrying cargo, the recognized standard of sixty revolutions per minute, so indelibly stamped on marine engineering practice, is unfavorable to the greatest economy. It is true that the absurdly bluff run aft which is found in many steamers interferes with a high number of revolutions; but there is no reason why a considerable advance should not be made. Engineers are often severely handicapped and sometimes utterly baffled by the design of the cargo "receptacle" which has to be propelled; and should the speed not reach the unreasonable expectations formed, the general inefficiency of the engines is at once set down as the cause, although the ship may have a displacement coefficient as high as 0·77 and may never run a straight course except by accident. That ships having a large cargo carrying capacity and traveling at a low speed are pecuniarily successful is not to be denied; but that beneficial results would follow from a slightly less box-like form is abundantly evident, though not yet universally appreciated in practice.

Sequence of Cranks.—In a triple engine some diversity of opinion seems to have existed as to the order of sequence for the three cranks. Many engines are arranged with the high pressure crank leading, followed by the intermediate crank, and the low pressure crank last; which causes excessive variation in the receiver pressure, increased ranges of temperature in the cylinders and considerable differences in initial stresses. The better sequence is high pressure leading, low pressure following, and intermediate last. Expanded diagrams were taken from an engine built on the Tyne with the common arrangement of cranks, namely, high, intermediate, and low: here the pressure in the receiver into which the high pressure cylinder exhausts varies as much 22½ lb., this large amount being probably due to the receiver being too small. The temperatures and stresses in the high pressure and intermediate cylinders approach equality; but the amount in each case is much greater than it would have been had the low pressure crank followed the high pressure. Other diagrams taken from an engine built in Scotland show the same sequence of cranks, namely, high, intermediate, and low; but the receiver from the high pressure cylinder being here larger, the variation of pressure in it is reduced to 12 lb. In this engine, however, the temperatures and stresses are astray, the effects of the latter being shown in the crank-shaft diagram. In the next diagram the low pressure cylinder follows the high pressure, with the evident beneficial effect of reducing the variation of pressure in the first receiver to 6 lb., and also reducing the ranges of temperature in the cylinders, as well as the initial loads on the pistons.

Number of Cranks.—The question of the most suitable type of marine engine resolves itself into two general divisions, namely, engines working on two cranks, and those working on three. Considerable difference of opinion has existed as to which is the better, and many engineers have decided in favor of the two-crank tandem, the high pressure cylinder being placed over either the intermediate or the low pressure. A preferable arrangement is to place two cylinders on each crank. But both these plans present very objectionable features. In a three-cylinder tandem it is impossible to obtain anything approaching equality of temperatures, stresses, and powers; and therefore the result is a loss of efficiency. In converting compound into triple expansion engines, the method of adding another cylinder on the top of the high pressure or low pressure is certainly an easy way of applying the triple expansion principle, but for obvious reasons it is a very objectionable one. In the diagrams from a tandem engine, in which every possible care was taken in the design, the results do not compare at all favorably with those from a good three crank engine. To have two cylinders on each crank is, undoubtedly, the best design for a two crank engine on the triple expansion principle, as it is then possible to get an approximately equal initial stress on each crank. This arrangement, of course, necessitates one of the three stages of expansion taking place in two cylinders, instead of in one. The diagrams from this type of engine show the range of temperature, varying from 59° to 81°, while the crank-shaft diagram is even worse than that from an ordinary compound on two cranks. A marine engine should be so designed that any working part can be easily examined or removed, and this is impossible with a tandem engine.

The arrangement of cylinders on three cranks fulfills the required conditions more nearly than any other design; and that this is appreciated by many engineers and shipowners is evident from the rapidity with which the three crank engines are entirely displacing the two crank. A few of the principal advantages are: (1) More uniform strain on shafting; (2) adaptability for a higher rate of revolution and increased piston speed, thus obtaining increased efficiency from the steam in the engine, as well as lighter machinery in proportion to the power developed; (3) less wear and tear; (4) easier accessibility of working parts; (5) interchangeability of parts, thus minimizing the consequences of a breakdown; (6) greater facility for repairs; (7) easier adjustment of temperatures, stresses, and powers. All these features are deserving of investigation.

Regarding the more uniform twisting moment on the crank shaft of a three crank engine, it is often advanced that the difference between crank-shaft diagrams from a two crank and a three crank engine is not so great as to cause any appreciable effect. It must be remembered, however, that the variable twisting strains on the crank shaft are reflected to a greater or less extent throughout all the working parts of the engine; and it seems certain that the beneficial influence of three cranks is far beyond what is generally accepted. There seems to be an impression that the length of an engine room is increased by the introduction of three cranks. But by placing the valve casings at the side, thus allowing the intermediate and low pressure cylinder covers to be brought close together, the total length of a three crank engine is made no greater than in nine-tenths of the existing compounds. In the Orient liner Lusitania, since

she has been converted to three cranks by the writer, the distance longitudinally over the cylinders is one foot shorter than before, and the indicated horse power is 800 greater.

Valve Gear.—The arrangements for working valves are becoming very extensive, and a review of the various devices which are employed to distribute the steam in modern engines would by itself occupy a very lengthy paper. The requirements of a good valve gear are that it shall give at both ends of the cylinder an equal distribution of steam at all grades of expansion, with a minimum of working parts and with no undue strains. The four principal methods adopted to work valves are: first, by the single eccentric; secondly, by the double eccentric; thirdly, by taking the motion from the connecting rod; and fourthly, by a compound motion derived from both the piston rod and the connecting rod. All four of these have their advantages and defects, and vary considerably in complexity and in multiplicity of parts. The single eccentric valve gear, giving almost perfect steam distribution, having few working parts, and being independent of the connecting rod or piston rod, seems eminently suitable to fulfill all the desired conditions. A sliding block gear of this kind has given every satisfaction in actual working; and the arrangement is such as to allow free access to all the working parts, while occupying a minimum of space. The end of the eccentric rod reciprocates on guide bars, which are inclined at various angles to suit the desired action of the valve. Motion is imparted to the valve spindle from an intermediate joint in the eccentric rod, which moves in an approximately elliptic path. A slot in the tail of the guide bars admits of varying their inclination to different extents for a given movement of the reversing engine, thus altering independently the cut off in each cylinder. The long leverage and easy motion reduce the wear and tear to a minimum, as results in actual practice have proved. An objectionable feature, however, in an engine fitted with this sliding block gear is that the valves are at the front, over the starting platform; and the exhaust has to be led by a belt round the low pressure cylinder to the condenser. To overcome this objection of the sliding block gear, a swinging link gear was designed. The eccentric rod, as in the last case, is placed diagonally over the condenser, but is here guided in an arc of a circle, by suspending it by a swinging link, centered on a pin, which pin is adjustable by the reversing engine into various positions for varying the grade of expansion either ahead or astern.

The movements for working the valve are transmitted from a joint at the end of the eccentric rod by a compensating link connecting the joint with one arm of an oblique lever, of which the other arm is jointed to the valve spindle. The compensating link is an essential and distinguishing feature of this gear; it is so placed and proportioned relatively to the other parts as to produce practically equal port opening and cut off at each end of the stroke. There is a quick and a slow movement of the valve at each end of its travel; the slow movement being at the maximum port opening, and the quick movement at the cut off. The lead is also constant at all grades of expansion.

Practical Results.—In the engines of the screw steamer Para, belonging to Messrs. Steel Young & Co., which made her maiden voyage to the River Plate about 3½ years ago, the cylinders are 19 inches, 35 inches, and 53 inches in diameter, with 33 inch stroke. These being the writer's first triple expansion engines, special arrangements were made for ascertaining the actual coal consumption per indicated horse power. Although, in order to get a correct estimate of an engine's performance, the quantity and temperature of the feed and circulating water ought of course to be considered, yet there are so many practical difficulties in the way of getting these particulars on board ship that it has been found impossible to obtain any reliable data on these points. This steamer still continues on the same run, averaging 9 knots an hour on 10¼ tons of coal; and has not yet cost anything beyond the usual overhaul for repairs. The great saving in coal consumption with the triple engines is apparent when comparison is made with two sister ships fitted with compound engines, the Ingram and the Wandle, belonging to the same company, and built by the same builders; the comparison is shown in Table I., which gives the average working over a period of three years.

TABLE I.—*Comparative Results from Three Similar Steamers with Compound and with Triple Expansion Engines.*

Name of screw steamer.	Ingram.	Wandle.	Para.
Length, feet and inches	257 6	257 6	257 6
Breadth, feet and inches	34 6	34 6	34 6
Draught, feet and inches	18 10½	18 6½	19 4
Dead weight carried, tons.	2,310	2,303	2,398
Type of engines	Compound	Compound	Triple
Boiler pressure, lb. per square inch	75	75	150
Speed, knots per hour	8½	8½	9
Indicated horse power, total	570	580	630
Coal consumption per day, tons	13¾	14	10¼

The diagrams are an expanded set taken during the three days' trial of the Para; but owing to the intermediate cylinder being rather too large, the equalities of temperatures and of initial stresses are disturbed; and the drop of the steam pressure in the intermediate cylinder is excessive, in consequence of the steam velocity being too great.

In Table II. is given a comparative statement of results from an approximately similar trio of boats in the same trade and under the same management on a round voyage to Java under average conditions. The triple engines in the Jacatra are of the same general design as those in the Para, but of greater power; the compounds in the two other boats are of the ordinary type.

TABLE II.—*Comparative Results from Three Steamers with Compound and with Triple Expansion Engines.*

Name of screw steamer	Fellinger.	Padang.	Jacatra.
Length, feet and inches	286 0	300 0	314 0
Breadth, feet and inches	35 0	37 0	37 9
Draught, feet and inches	20 3	21 9	21 6
Dead weight carried, tons	2,600	3,000	3,300
Type of engines	Compound	Compound	Triple
Boiler pressure, lb per square inch	70	78	140
Speed, knots per hour	9	9½	10
Indicated horse power, total	600	700	800
Coal consumption per day, tons	15½	18	13½
Ditto per I.H.P. per hour, lb.	2·19	2·13	1·41
Quality of coal used.	German	Cardiff	Mixed

The design of the triple engines of the Union Company's screw steamer African and Messrs. Glover Brothers' screw steamer Shakespear differs from that in the Para and Jacatra in having the valves situated over the condenser, thus doing away with the exhaust belt around the low pressure cylinder, and giving a free, open front to the engines; all the working parts are thereby rendered easily accessible, with every facility for overhauling. The expanded diagrams taken from the Shakespear under average working conditions have already been referred to. The equalities of temperatures, initial stresses, and powers are as near as it is practically possible to get them; and this result has been obtained by carefully considering the various points which influence the efficiency of the engine. The expanded diagrams taken on the trial trip of the African in Stokes Bay show 1,086 horse power, developed at 85⅛ revolutions per minute. They thus prove that there is a sacrifice of weight in triple engines when making no more than 60 revolutions per minute, inasmuch as the power developed at 66 revolutions per minute by the duplicate engines of the Shakespear is much less, namely, 871 horse power. The expanded diagrams from the original compound engines of the Union Company's screw steamer Anglian were taken under average conditions on a voyage to the Cape. The mean consumption of coal per day over eight voyages was 24 tons, or about 2·1 lb. per indicated horse power per hour. The compound engines were of the ordinary two cylinder type, the valves being directly over the screw-shaft, and driven by the usual link motion. Diagrams were taken on a trial trip after the engines had been converted to triple expansion; and on her voyage to the Cape, the average speed being exactly the same as in the eight voyages above referred to, the coal consumption was one-third less, namely, 16 tons; and as she is placed on a foreign station where the cost of fuel is about £2 per ton, the economy of the conversion is obvious. The method of converting consisted in replacing the old high pressure cylinder by a new intermediate cylinder, and adding a new high pressure engine complete on the forward end of the screw-shaft, the high pressure and intermediate valves being driven by sliding-block gear. By arranging the valve casings at the side of the high pressure and intermediate cylinders, the distance fore and aft over the present engines is very little more than before, although the power is now sufficient to drive the vessel one knot an hour faster than her former maximum speed. The original compound engines of the Orient Company's screw steamer Lusitania were of the two cylinder type, with an expansion valve on the high pressure cylinder. The diameters of the cylinders were 58 and 103 inches, with a stroke of 4 feet; and the boiler pressure was 55 lb. per square inch. The diagrams were taken under ordinary working conditions on a voyage from London to Sydney, the average daily consumption being 52 tons of Welsh coal. The old cylinders were afterward replaced by new ones, and a high pressure engine was added complete, its valve being worked by swinging link gear. By arranging the intermediate and low pressure slide valves between the intermediate and low pressure cylinders, the old gear was utilized; and by reducing the valve casings to modern proportions, the distance lengthways over all the three new cylinders is one foot less than before. Subsequent diagrams represent the present average working conditions of the engines, the increased power propelling the vessel at a much higher speed. The coal consumption at this power is about 50 tons per day; but traveling at the former speed, the consumption is reduced from 52 tons to 37 tons; so that, allowing eighty steaming days for a return voyage to Australia, the saving of coal is 1,200 tons, while the cargo-carrying capacity is largely increased. The foregoing results represent a fair average of those obtained from thirty triple expansion engines, all of the three crank type, which have been designed by the writer during the last three years; and are sufficient to prove that this kind of engine is most efficient, and is undoubtedly the engine of the future.

Artificial Draught for Boilers.—The next step in marine engineering is probably the application of artificial draught to boiler furnances. Expanded diagrams were taken from the engines of the screw steamer Stella, belonging to Messrs. Herskind & Woods, between Hartlepool and Dover on her voyage to Bombay; and as a steady boiler-pressure is always maintained, the engine power may be considered as constant. The average speed between port and port from Hartlepool to Bombay was fully 9 knots per hour, the dead weight carried was 3,680 tons, and the daily consumption of North Country coal was 13·6 tons. This performance is very remarkable, when the dimensions of the ship are considered, namely, length 302 feet, breadth 38 feet, and displacement coefficient as high as 0·77. The special features of the arrangement are the method employed for heating the air both outside and inside the uptake, and the application of balanced fire-doors, which on being opened shut off automatically the hot air supplied by the fan, both above and below the fire bars. This is the first application of artificial draught to the boilers of triple expansion engines; and should the results fulfill expectations, there is little doubt that the plan will be extensively adopted, as being yet another step toward economy of fuel.

Mr. Morrison (Mr. Wyllie's partner) said that the indicating diagrams had been taken by three operators at practically the same instant, so as to minimize the possibility of error due to the fluctuations of steam pressure in the boiler and the two receivers. He had laid on the table the actual diagrams taken on the trial of the Para, from which the members would see their true relative forms. He had also brought the actual diagrams of China tea steamer Kaisou, which made a very economical voyage last season. As a proof of the correctness of the results obtained by the Stella, he had on the table the engineer's abstract log, showing the total coal consumed on the voyage, and also the daily steer of the ship. She was now on her second voyage, and according to latest advices was traveling at an average speed of 8½ knots on a daily consumption of 14 tons of very inferior coal. He agreed with a remark made at the British Association that the economy resulting from the application of artificial draught must be to obtain higher evaporative efficiency with good coal, and not to attempt to consume rubbish containing about 40 per cent. of clinker. The all important factor of high pressure to which Leeds might be proud to contribute, namely, Fox's corrugated flues, had been fitted in all the high pressure boilers for the thirty triple expansion engines designed by Mr. Wyllie, and in no instance had those furnaces given trouble.

Mr. W. Parker (Lloyd's), after expressing his great regret at the loss of Mr. Wyllie, said he believed it was acknowledged by all steamship owners that the compound engine was now extinct. He might mention the case of two large P. and O. steamers built under his inspection with exactly the same dimensions and the same power, one propelled by a triple expansion engine working 45 lb. per square inch, and the other by an ordinary compound engine working 90 lb. per square inch. Both of them steamed at the same speed, twelve knots an hour, to Australia, and the vessel with the triple expansion engine burnt 1,200 tons less coal than that with the compound engine. In the case of two other vessels, one was fitted with a compound engine working at 90 pounds per square inch, carrying 3,000 tons cargo. She steamed ten knots an hour, and burnt 20 tons of coal per day. The second vessel was fitted with a triple expansion engine working 150 lb. per square inch, and she made her voyage to India in the same length of time as the other, burnt the same amount of coal, and carried 4,200 tons of cargo, or 1,200 tons more than the other. Other cases might be cited of a similar nature showing that the triple expansion engine was really the engine of the future. There were now 150 pairs of triple expansion engines running, giving results similar to those which he had mentioned. Practical experience, he thought, showed that four cylinders were better than three. He agreed that the intermediate and low pressure cylinders should be steam jacketed, but unfortunately jackets were seldom properly attended to in marine engineering. As to the question of three cranks or two cranks, there was, he believed, a slight difference in favor of three cranks, but the two-crank engine could be made to work almost as uniformly. He had been recently told by the secretary of one of the largest mail companies that the engines altered by Messrs. Richardson, of Hartlepool, had given so much satisfaction that the company had determined to alter the whole of their fleet, finding that the difference in consumption effected by the alteration really meant the dividend. He was pleased to see present Mr. James Riley, of the Steel Company of Scotland, whose production of mild steel had enabled those changes to be brought about, and Mr. Sampson Fox, of Leeds, to whose energy and perseverance they were greatly indebted.

Mr. Charles Cochrane considered that the paper was a most valuable one, and he was very sorry that the author was not present to hear the comments made upon it. It must be clear to all that a pound of coal was capable of evaporating various quantities of water, according to the boiler in which the combustion of that coal was effected, and he could not help feeling that the paper ought to be divided into two separate parts, one having relation to the boiler duty and the other having relation to the engine duty. In discussing a matter of this kind before such an institution, they ought to have some more scientific standard than that of pounds weight of coal consumed per horse power per hour. Was such a standard to be obtained? He contended that there was, and one to which any calculators of the kind might be referred in future—namely, to the total weight of steam used per indicated horse power per hour. In that way, and that way alone, could they detect the faults of the engine and know whether it was doing its proper duty. The conditions of boiler which interfered with its evaporative power, such as the temperature of water introduced into it, ought not to be mixed up together; and if they were to discuss the merits of the engine on scientific data, they ought to adopt that standard of reference in future. He would venture to make a further observation, which was that where Mr. Morrison found the velocity of steam had really shown itself in the wire-drawn condition of the diagram, he had the elements of ascertaining what should be the lineal velocity of the steam when getting the full expansion out of it. Knowing the size of the ports which had failed to give him perfect satisfaction, he had a grand opportunity of ascertaining at all the varying conditions of pressure what was the lineal velocity of steam, which might be useful for every one when they were deciding on the size of their steam ports in any engine that was being constructed.

Professor Kennedy protested against three cylinder engines not being classed as compound. He wished to join in Mr. Cochrane's request that, if possible, Mr. Morrison would let them know the size of the ports of the engines. If that could possibly be given, the Institution would be still more indebted to Mr. Wyllie for the very large amount of information which he had put before them in such a concise form. The question of triple expansion engines had been a good deal before them lately, but he regretted that they had not yet got any exact experiments of any kind on marine engine economy. They had statements of averages, but those could not be entirely trusted, and he hoped that in the immediate future it might be possible to have some more accurate data to go upon. He thought it was not too strong a thing to say that the indicator cards did not show more than 75 per cent. of the steam, and those who had practically measured these things would bear him out in what he said. In the triple expansion engines the high pressure compression was very well carried out, and the high pressure clearance was very small, but the fall of temperature was about the same as in a two cylinder engine, and there were no jackets. In the Para the evaporation was stated to be 12 lb. of water per lb. of coal; in the Anglian, 13·6 lb.; in the Stella, 13·9 lb.; and in the Lusitania, 16 lb. He did not hesitate to say that that was impossible with ordinary fuel. From 12 lb. to 14 lb. of water evaporated per lb. of coal was about as much as could be got in a calorimeter, and in the case mentioned there was necessarily an imperfect combustion to begin with, and he believed that 10 lb. of water evaporated per lb. of coal was very good, and he should receive a statement of 11 lb. with very great reserve, and 12 lb., 13 lb., 14 lb., or 15 lb., he could not believe. Such figures were not consonant with anything that was known about the evaporative power of fuel. He, however, did not wish it to be supposed that he disagreed with the author's main contention that three cylinder engines were better than two cylinder engines in particular cases, and the reason of the advantage in those cases was quite clear. It was legitimately obtained according to the diagram by using higher pressure,

and there was no reason on paper why it should not be equally well done by two cylinders. It did not prove that three cylinders were better than two, but simply that it was better to use 150 lb. pressure than 50 lb.' There were important practical questions which might make it inconvenient to use the high pressure in two cylinders, and there were also reasons for not doing so connected with the range of the temperature, with initial condensation, etc. He believed that inefficiency in the low pressure cylinder was caused mainly by defects of passages, by defects of receiver, etc., and could be remedied by alterations outside the low pressure cylinder itself. Large economies should be looked for rather in improvement in high pressure cylinders than in low pressure cylinders at present.

Mr. Mudd (Hartlepool) thought that the thanks of the Institution were specially due to Mr. Wyllie at this time, when every one was engaged in the design and manufacture of triple expansion engines, working out his own ideas, and feeling his way through the practical difficulties which higher pressure and higher temperature had brought into existence, and when the principle of triple expansion was only just emerging from the experimental stage. Indeed, he thought that the paper ought to be regarded as a kind of legacy left to the Institution and to the profession at large. The author stated that the most important conditions to be considered in order to obtain an efficient engine were that there should be approximate equality, first, in the range of temperature in each cylinder; secondly, in the initial strength of each crank; and thirdly, in the indicated horse power of each engine. That was much too general a statement; at all events, he was not willing to admit it without considerable additions and limitations. It was either too wide or too narrow a statement to correspond with the facts. It was possible for an engine to comply precisely with those conditions, and at the same time to be working most inefficiently; and therefore it was clear that those were not the whole of the conditions that determined the efficiency of the engine. If they were the test of efficiency in a triple expansion engine, they must also be so in an ordinary compound engine and in a single engine, but the test would utterly fail in this last case. The efficiency of a triple expansion engine, like that of every other engine, was dependent on a large number of conditions which might be classified under two heads, namely, those which affected the efficiency of the steam and those which affected the efficiency of the mechanism. It seemed clear from other parts of the paper that the author intended only to allude to the question of efficiency of the steam, and not to the efficiency of the engines as a whole. The range of temperature in the cylinder was the one point of the three named by Mr. Wyllie which had a direct bearing on the efficiency of the steam. With regard to steam jackets, he wished to say that if any one cylinder was to be jacketed, it should be the low pressure cylinder, although specifications frequently stated that high pressure cylinders were to be jacketed.

Mr. D. Adamson said he had had some experience in regard to triple and quadruple action engines many years ago, but he had had no experience in regard to marine engineering. He had expressed his opinion that, instead of continuing the use of one cylinder, they ought, now that they had large engines and large shafts, to distribute the percussive force over many crank pins, and he was glad that Mr. Wyllie had brought the subject before them so prominently. He had several structures working at 250 lb. pressure. If they went beyond that, he thought a quadruple action engine would be required. There was something yet for young engineers to do in improving the economy of steam engines. He believed that dividing the force over three or four crank pins was much better than running the risk of breaking the shaft by putting the whole force on one. He was convinced by experience that a quadruple engine would beat the triple cylinder immensely. He was not alarmed at the irregularities pointed out by Professor Kennedy. The cards might be a little wrong and irregular, but they were the best means they had of getting the results. Mr. Parker's observations had demonstrated the real economy of the triple engine. He had had some experience of high pressure steam. He had 3,000 boilers working at very high pressures, nearly all of steel, without having had a single explosion. His own experience was that metals did not suffer until they got to 500°. With a comparatively pure metal they might have 550° temperature before it became dangerous, or even 600° in the case of a pure iron with no alloy. He thought there was a prospect of 200 lb pressure at least being obtained by the multiple system. To work the triple action efficiently, the system of triple cranks should be adopted.

Professor Smith, of Birmingham, said he thought that after the criticisms of Professor Kennedy, they required some further explanations as to the calculations made by the author. He was quite willing to believe in the evaporation of 9, 9¼, or 9½ lb. of water per lb. of fuel, but he was very skeptical with regard to 10 lb. and upward. He thought that the greater efficiency of the triple expansion engines was chiefly due to the possibility of using a very much higher initial pressure, but the question still remained as to whether they ought to have more than two cylinders. In order to distinguish between the efficiency of the boiler and of the engine, it was only necessary to give separately the mechanical horse power of the boiler and the horse power of the engine, to compare the one with the other, and it would generally be found that the engine horse power was greater than the mechanical horse power of the boiler.

VOUSSOIR CALCULATION AND MEASUREMENT.

By C. POWELL KARR, C.E., Consulting Architect, New York.

AN arch of masonry consists of a ring of wedge-shaped stones, called arch stones or voussoirs, pressing against each other at surfaces called bed joints, which are or ought to be perpendicular or nearly so to the soffit or internal concave surface of the arch. The outer or convex surface of the ring may be either a curved surface parallel to the soffit or a series of steps to sustain the vertical pressure of that part of the load which is superimposed upon them. The projection of the soffit upon a vertical plane is the curved line of the arch, and is called the intrados. The upper limiting line of the arch stones, whether it be parallel to the intrados or whether it be made up of the broken horizontal-vertical line of the steps, is called the extrados.

The highest point of the intrados is the crown. The line or bed from which the arch springs is the spring line. If an arch be built in a wall connected with other work, the supporting beds of the arch are called its abutments. The supports to two successive arches are called piers. The sides of the arch are called the haunches or reins. The vertical walls included between the haunches and an imaginary line passing through the spring line parallel to a line passing through the extrados above the keystone are called the spandrels. The supporting blocks, or the two voussoirs at the spring line, are the skewbacks.

In addition to their own weight, arches usually support a permanent or moving load or surcharge of earth or masonry or both. Suppose an arch were divided vertically at the crown by a plane at right angles to the span into two equal homogeneous semi-arches. Suppose one of these semi-arches to be removed, and it were possible to ascertain the force which is just necessary to keep the remaining semi-arch in its place. This force is the thrust of the arch, and is called the horizontal thrust. It is equivalent to the mutual pressure which exists between the two semi-arches in position to keep them in equilibrium. The total thrust of the arch is the sum of the vertical and horizontal thrusts, and in an equilibrated arch this total thrust is transferred to the abutments or piers. The limits of this article will not permit of an inquiry into the theory of the arch or the principles of voussoir design, and we can only make a passing reference to them.

In a general way, the thickness of an arch is assumed to correspond with some precedent or, in the absence of such, with the results of some empirical rule; then the load which it must sustain is calculated, the location of the line of resistance is plotted graphically or calculated analytically, and the thickness of the arch is so adjusted that the centers of resistance of the voussoir joints shall fall within the middle third space of the depth of the arch ring. A most unscientific method, it is admitted, but it is the one which is generally followed. The thickness of the arch is the criterion for the depth of the keystone. Captain Woodbury's special rule for designing an arch is the safest guide. It is: Depth of arch at the key in feet is twice the actual thrust acting one-third of the length of the joint from the extrados divided by the pressure (P) per unit of surface at the upper edge of the keystone joint. Captain Woodbury also remarks that in light arches the thickness at the key is found to vary from one thirtieth to one thirty-third of the span, and we will call this his general formula. The writer's observations and study of existing structures would lead him to ask of engineers a modification of Mr. Trautwine's formula for circular and segmental as well as elliptic arches by changing the constant in the denominator to 3·5 instead of 4, as Mr. Trautwine has it. We will present a table of comparisons. My rule is—

$$D = \frac{\sqrt{\text{radius} + \text{half span}}}{3{\cdot}5} + 0{\cdot}2 \text{ ft.}$$

TABLE OF DEPTH OF KEYSTONES BY EMPIRICAL FORMULÆ.

Edinburgh Bridge, by Mylne. Span, 72 ft.; rad., 36 ft.; depth of key, 2·75.

	Ft.
By Perronet's rule	4·20
Formula: $D = 13 \text{ in.} + \frac{\text{span}}{23}$	
By Trautwine's rule	2·32
Formula: $D = \frac{\sqrt{\text{radius} + \text{half span}}}{4} + 0{\cdot}2$ ft.	
By Rankine's rule	2·07
Formula: $D = \sqrt{0{\cdot}12 \times \text{radius}}$, or $D = \sqrt{0{\cdot}17 \times \text{radius}}$ for arch series.	
By Woodbury's rule	2·40
Formula: $D = \frac{S}{30}$, or $D = \frac{2 \times \text{horizontal thrust}}{P}$	
By Hurst's rule	2·54
Formula: $D = C\sqrt{\text{radius}}$. $C = 0{\cdot}3$ for stone; $0{\cdot}4$ for brick.	
By Karr's rule	2·62
Formula: $D = \frac{\sqrt{\text{radius} + \text{half span}}}{3{\cdot}5} + 0{\cdot}2$ ft.	

Chestnut St. Bridge, Philadelphia. Span, 60 ft.; rise, 18 ft.; radius, 34 ft.; actual depth of key, 2·50 ft.

By Perronet	3·68 ft.
" Trautwine	2·20 "
" Rankine	2 "
" Woodbury	2 "
" Hurst	3·10 "
" Karr	2·48 "

Dean Turnpike Bridge, near Edinburgh, Scotland. By Telford. Segmental arch; span, 90 ft.; rise, 30 ft.; radius, 48·90 ft.; actual depth of key, 3 ft.

By Perronet	6·05 ft.
" Trautwine	2·62 "
" Rankine	2·88 "
" Woodbury	3 "
" Hurst	2·10 "
" Karr	2·97 "

Wissahickon Bridge, Philadelphia. 65 ft. span; 23 ft. rise; depth of arch stones, 3 ft.; radius (by calculation), 57·42 ft. Series of arches.

By Perronet	3·90 ft.
" Trautwine	2·57 "
" Rankine	3·12 "
" Woodbury	2·17 "
" Hurst	2·27 "
" Karr	2·90 "

It will be seen, upon comparison, that the proposed modification of Mr. Trautwine's rule is much nearer, upon the whole, than his own or any of the others. In justice to Mr. Woodbury, the writer would say that the figures opposite his name are derived from a formula founded upon the observations of that noted writer, which we call for distinction his general formula, and that a formula which depends for its factors upon the thrusts as determined from the actual loads which the arch must bear, or the strains it must meet, would be more in accordance with the best practice, and the truth, than to try to rely upon any empirical formula, no matter how near the truth such a formula may *seem* to be. Captain Woodbury's special rule, as first given, is more in accordance with this principle, as it is partially founded upon the forces in action to maintain equilibrium; but it could not be applied in our comparisons, because the loads which the various bridges have to sustain are not to be had readily, and, therefore, two of the elements of his formula could not be obtained; but in designing an arch, these, with the exception of the point of application of the horizontal thrust, are the known elements upon which the arch is to be based, and the formula can therefore be used. Whenever the word "crown" is used in this article, the nomenclature of Prof. Rankine will be adopted, viz., the "crown" is the highest point of the intrados, not of the extrados, as some writers use it. These formulæ should only be used as clews to the discovery of the depth of the keystone, and therefore of the arch itself, for the final determination should be made by means of the graphical method of determining the line of resistance, and proportioning the depth of the arch to the depth of the load it is obliged to sustain.

THE PRINCIPLES INVOLVED IN CALCULATION.

For the purposes of preliminary calculation, the only part of a voussoir we need to draw or know is its face. An examination of all voussoir calculation will discover everywhere a constant reference to and application of the relations of the different parts of a triangle to each other. It need not always be a right-angled triangle, but in the analytical method it generally is. Given two parts of a right-angled triangle, we can solve all the rest. In doing so we call to our aid the formulas obtained from plain trigonometry. In every right-angled triangle, the two sides adjacent to each other which include the right-angle between them are considered, the one side the base and the other side the perpendicular, and it does not matter which side is considered the perpendicular or the base, the side opposite the right angle is the hypotenuse. The angle included between the base and the hypotenuse is called the angle opposite the perpendicular, or the angle at the base. It is the angle which is the most constantly used in obtaining the value of the perpendicular. It is not necessary to know all the formulas which pertain to the right-angled triangle in order to solve the unknown side or angle. The formulas in general and their derivatives are as follows:

1. The perpendicular is equal to the hypotenuse into the sine of the angle at the base, or $P = H \times \text{sine of angle at the base}$; also, $\text{sine} = \frac{P}{H}$

(2.) The perpendicular is equal to the base into the tangent of the angle at the base, or $P = B \times \text{tang. of angle at the base}$; also, $\text{tang.} = \frac{P}{B}$

(3.) The perpendicular is equal to the square root of the hypotenuse squared minus the square of the base, or $P = \sqrt{H^2 - B^2}$

(4.) The base is equal to the hypotenuse into the cosine of the angle at the base, or $B = H \times \text{cosine of angle at the base}$; also, $\text{cosine} = \frac{B}{H}$

(5.) The base is equal to the perpendicular divided by the tangent of the angle at the base, or $B = \frac{P}{\text{tang.}}$

(6.) $B = \sqrt{H^2 - P^2}$

(7.) $H = \sqrt{P^2 + B^2}$

(8.) The hypotenuse is equal to the perpendicular divided by the sine of the angle at the base, or $H = \frac{P}{\text{sine of angle at base}}$

(9.) The hypotenuse is equal to the secant of the angle at the base into the base, or $H = B \times \text{secant}$.

From geometry we have:

(10.) Two triangles which are mutually equiangular have their homologous sides proportional, and conversely.

(11.) Two triangles are similar when they have their homologous sides parallel each to each or perpendicular each to each.

(12.) In a right-angled triangle, if a perpendicular is drawn from the right angle to the hypotenuse: 1. The triangles on each side of the perpendicular are similar to the whole triangle and to each other. 2. The perpendicular is a mean proportional between the segments of the hypotenuse. 3. Each of the sides is a mean proportional between the hypotenuse and its segment adjacent to that side.

Wherever similarity between two triangles can be detected, the discovery is rewarded by a summary calculation. If you are not familiar with the methods of graphical calculus, borrow an idea from the economy of that beautiful science. Its fundamental basis of comparison is the triangle, and the product of lines and areas is effected by means of the properties of similar triangles. Wherever similar triangles can be brought in, either by a constructed line or otherwise, to aid in the calculation, the solution can at once be proceeded with, and its accuracy and celerity is astonishing. By arbitrarily fixing the lengths of the chords of the voussoirs, as may be done sometimes, an entire set of voussoirs may be calculated without reference to the value of more than one angle; and sometimes, under favorable conditions, even that may be dispensed with, as shown in our first and simplest calculation. Endeavor, therefore, to divide the voussoir faces in such a manner by parallel lines or other constructions so as to establish symmetrical relations, so that the quantities involved can be thrown into a proportion, and the unknown term solved at once. In the solutions given, we have in every case used the values of natural sines, tangents, and cosines which are the values of an arc whose radius is unity, as fewer mistakes by the novice are likely to occur than in

using the logarithmic sines, etc., unless the latter are in constant use. When the value of the radius is a whole number, the natural method is the shortest.

The student will at once perceive from the calculations and the drawings that no allowance has been made for joints. Whatever joint thickness has been decided upon, such thickness is appended in a footnote to the drawing, and the stonecutter must make his own reductions accordingly. Thus, if the instructions in the footnote read, "All joints to be three-eighths of an inch at most," the stonecutter must subtract three-sixteenths of an inch from each face except the soffit face, in order to get the exact dimensions of the face. Where the section of a shaft intersects a tunnel, and voussoirs have to be constructed to fit into the curve of intersection, the voussoir stones will be of such a shape, having two curved faces not parallel to each other, that it will be impossible to convey to the stonecutter by means of elevations or sections an adequate idea of the contour of the voussoir. In such a case resort must be had to isometric perspective views from various standpoints, and in extremely difficult cases resort must be had to miniature models of wood

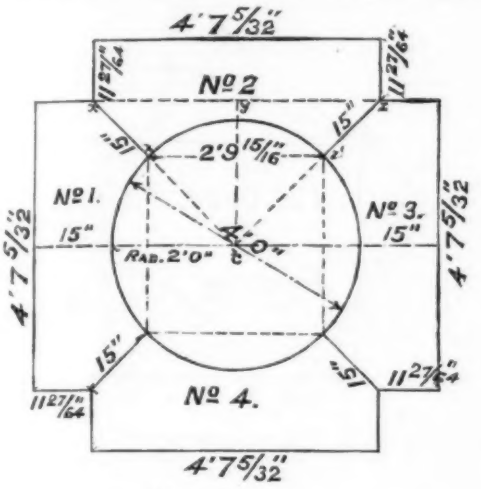


FIG. 1.

PLAN OF HORIZONTAL VOUSSOIRS.

made to the scale preferably of one inch equals one foot.

In order to present the subject in a simple and, at the same time, comprehensive manner, we have selected for our illustrations from actual construction a plain set of horizontal voussoirs, on account of their simplicity; then a set of typical vertical voussoirs, which are the most frequently met with. They are by no means to be taken as a model of design, for they are too heavy for such a small span, but, in their outline and the method of their construction, they exemplify the principles involved in an arch of greater span and more intricate construction. We have concluded the calculations by referring to a special case of horizontal voussoirs from actual construction, which involve some of the difficulties sometimes met with in voussoir calculation. Where it is possible to draw an arch to a scale of three inches equal one foot, and by using a beam compass, great accuracy and delicacy of line work may done, so that all the distances by construction can be scaled off on a decimal scale, the thousandths part of a foot can be read off, and no analytical work need be entered into if the drawing is accurately made; but this method is seldom pursued.

In Fig. 1 we have a plan of four horizontal voussoirs designed for the connection of a water tower shaft with its sewer outlet. The faces shown in the drawing form the bed joints for the superincumbent masonry, and

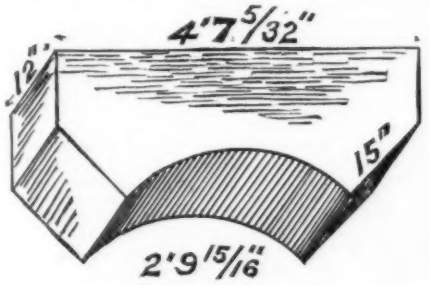


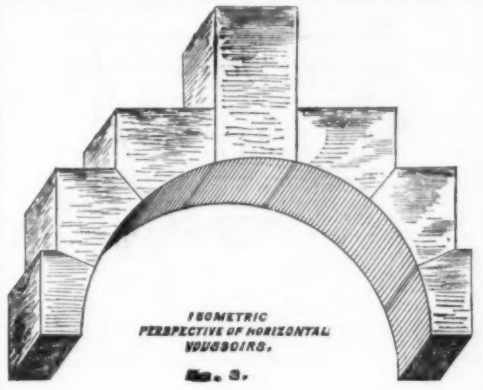
FIG. 2.

ISOMETRIC PERSPECTIVE

SKETCH OF STONE Nº 2.

therefore are not seen upon the completion of the well shaft masonry. The curved faces above are visible; see Fig. 2. In deciding upon the lateral dimensions of such stones, it is necessary to know what the horizontal pressure from without and within will be. The weight supported by the stone must also be ascertained. The vertical depth may or may not correspond with the depth of the adjoining course of masonry, as its depth may be governed by other conditions, which should be studied. The elements of voussoir design stand upon debatable ground, and the thickness of any given arch under every possible load is still an unknown quantity. We shall confine our remarks in this article to calculation and measurement, to pointing out the desirable pathway of method and simplicity to students and beginners, and we do not pretend to make an exhaustive exposition of the subject.

In the drawing submitted (Fig. 1), the data given are: 1st. The diameter of the shaft; 2d, the length of the radial joints and their number; and 3d, the width of each voussoir on the bed joint, taken on a line passing through the diametral section, such line being a continuation of the shaft diameter as shown in the drawing. At the start we are informed that the circumference of the shaft is to be divided into four equal parts by the intersection of the radial joints. This fact at once determines the uniformity in size of all four voussoirs. Then we know, from having the size of one stone, further calculation will be unnecessary. The method of procedure is as follows: 1st, to find the length of the chord subtended by the arc. The circumference is divided into four equal parts by the radial joints; lines joining these points, as shown in Fig. 1, will form the chords of the arcs, and also the side of the square which may be inscribed in the circle. This geometrical fact enables us to find the chord at once. To find the chord referred to the radius, we multiply the radius by 1·4142; or, referred to the diameter, by 0·7071. These factors are thus obtained: Suppose we have a circle whose radius is unity; inscribe a square within that circle. The diagonals of the square will intersect at the center of the circle, and are diameters of the circle. The semi-diagonals form two equal sides of a



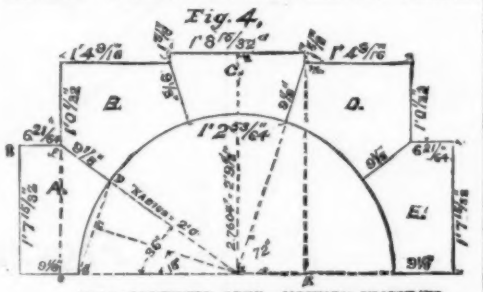
right-angled triangle, and are radii of the circle. The chord of the intercepted arc or one side of the inscribed square is the hypotenuse of this triangle. These remarks apply equally to each of the four triangles. On the principle of geometry that the square on the hypotenuse is equal to the sum of the squares described on the two other sides (see formula 7), we can solve the hypotenuse of any one of these right-angled triangles. Each chord of the intercepted arc is a hypotenuse. Let the radius be 1, then $1^2 + 1^2 = 2$, and $\sqrt{2} = 1{\cdot}4142$, which is therefore the chord expressed in feet of the intercepted arcs. Q. E. D. Thus we see that the factor of multiplication referred to the radius is 1·4142; or, referred to the diameter, which is twice the radius, is 0·7071. Upon inspecting the geometric figures inclosed by the chords and a line parallel to one of them through the outside edges of the radial joints (as shown in Fig. 1—$x\,z$), we perceive the triangles so inclosed are homologous, therefore the side of the smaller triangle; $c\,z' : c\,z :: z'\,x' : z\,x$ (see equation 10), or $2' : 3'\,3'' :: 2'\ 9\frac{1}{16}'' : x$;

$$4 \times 0{\cdot}7071 = 2{\cdot}8284' = 2'\ 9\tfrac{1}{16}'' \text{ hence } x = \frac{2{\cdot}8284 + 3{\cdot}25}{2} =$$

$4{\cdot}5961' = 4'\ 7\frac{5}{32}''$. The triangle $x\,c\,z$ is an isosceles triangle, the angle $x\,c\,z$ is a right angle, the line $c\,y$ bisects the side $x\,z$ and the right angle $x\,c\,z$; therefore, the angle $y\,z\,c$ is equal to $y\,c\,z$, both angles are 45°, and hence the side $c\,y$ is = to $y\,z$; but $y\,z$ is equal to ½ of $z\,z$ or to 2·2978'. By construction, the distance from the center of the circle to the outside edge is equal to 2' 0" + 1' 3" or 3' 3" = 3·25 feet; therefore the external unknown distance in the corners is equal to 3·25 feet minus 2·2978 feet, equal to 0·9522 foot, equal to $11\frac{27}{64}$ inches.

Summary of process: 1st. Ascertain the length of the chord. 2d. Form a proportion of the sides of the two homologous triangles to obtain the outer unknown side. 3d. Bisection of the calculated side to obtain the altitude of the large triangle. 4th. Subtraction of the altitude from the total distance of the outermost edge of the stone to the center of the shaft, to find the last unknown dimension required.

In Fig. 4 we show the elevation of a typical full centered arch. The arch is divided into five voussoirs

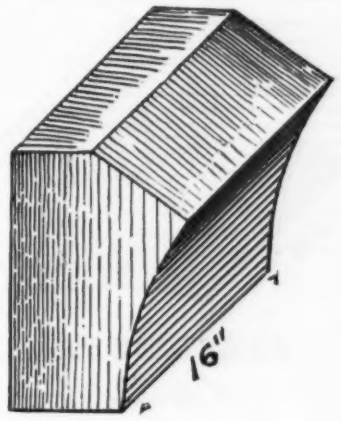


FULL-CENTERED ARCH—VERTICAL VOUSSOIRS.

ELEVATION.

having equal chords. As equal chords subtend equal arcs, and as equal arcs measure equal angles at the center, it follows that the angle 180° subtended by the full centered arch is divided into five equal angles, or 36° each.

In planning the dimensions of a set of voussoirs to an arch whose intrados and extrados have been determined by a previous calculation, the chords of an even number of arcs are arbitrarily assumed at some convenient readily measured distance, and are made equal throughout, their angular measure is subtracted from the total angular measure of the arch, and the remainder becomes the angular measure of the keystone, provided it is greater than the angular measure of the chord of one of the even number of arch stones; or the keystone chord is arbitrarily fixed, either by some empirical rule or by careful calculation from the conditions imposed upon the structure, and the remaining arcs divided into an even number of arcs where such a division will not conflict with the conditions of the structure; or the horizontal or vertical edges are assumed, radial lines are run to the intersections, and the radial joints and chords are calculated. In the plan before us, to simplify the calculation, we have assumed all the radial joints and chords to be respectively equal, joint to joint and chord to chord. For wall openings in ordinary buildings to be spanned by small arches, no serious difficulty will occur in permitting the adoption of radial joints to act as the bed joints of the voussoirs. Such a plan has been adopted in this case, but in large spans, as in some large bridges where a moving load is to be provided for, with the utmost economy of material in the structure, scientific forethought requires the bed joints to be approximately normal to the line of pressure. The voussoirs under consideration are called vertical voussoirs because they span a vertical opening, such as that of a doorway, a window, or a relieving arch in a vertical wall. The calculations required are the vertical and horizontal joints. In order to make a horizontal joint coincide with the bed joint of some horizontal course previously established, it is necessary sometimes to establish the intersecting vertical joint, or *vice versa*, but this is bad practice. The mason will



STONE-A- AND REVERSE

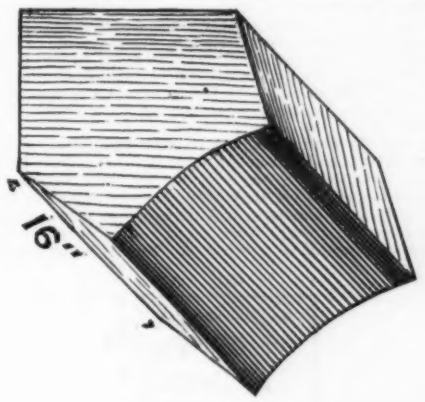
OF STONE E.

FIG. 5.

thank you to establish your horizontal joints of your voussoirs first and plan your courses to correspond, so as to avoid difficult stone cutting and poor stone laying.

We have assumed a simple case in order that the demonstration may be the more readily understood, and have avoided the use of logarithms in order to make the calculation plain to those who have had no time to learn their use and become familiar with their application. The principles involved are applicable to the solution of the most difficult problems in stone cutting.

The order of procedure will be as follows: 1st, to find the chord common to all the arcs, thus making necessary but one calculation; 2d, to ascertain all of the horizontal joints consecutively, beginning at the spring line of the arch; 3d, to determine the vertical joints in regular order, beginning at the spring line.



STONE-B-AND REVERSE

OF STONE-D.

FIG. 6.

1st. To find the chord of stone A (see Fig. 4), denoted by $d\,b$. We see by the drawing that the line $a\,c$, drawn perpendicular to the chord, bisects that chord; from the principle in geometry that the diameter which is perpendicular to a chord bisects that chord, and forms two right-angled triangles $a\,c\,d$ and $a\,c\,b$. The side $d\,c$ or $c\,b$ is equal to the side $d\,a$ or $b\,a$ multiplied by the natural sine of the angle opposite $c\,d$ or $c\,b$ (see formula 1). The angle opposite either of these sides, by reason of bisection, is equal to 18°, as the angle at the center of the triangle $d\,a\,b$ is 36°; hence $d\,c = d\,a \times$ nat. sine of $d\,a\,c$, and $b\,c = d\,c$; $d\,a\,c = 18°$; $a\,d = 2'\ 0''$. The natural sine of 18° is 0·309017. These quantities being substituted in the last equation, we find $d\,c = 0{\cdot}6180$ feet; but $d\,b = d\,c + c\,b$, therefore $b\,d$ is equal to 1·2360 feet, equal to $1'\ 2\frac{53}{64}''$. Q. E. D.

This chord therefore is the chord of each of the five arcs by the conditions given.

2d. In the triangle $a\,e\,f$, which is a right-angled triangle, we are to ascertain the distance $a\,e$. If we

know $a\,e$, we can find $e\,h$, which is equal to $f\,g$, by subtracting $a\,e$ from $a\,h$, a distance which is given, *i. e.*, $2'\ 9\frac{1}{8}''$.

$a\,e$ is the base of the right-angled triangle $a\,e\,f$, and we solve it by the formula that the base is equal to the hypotenuse into the cosine of the angle at the base opposite the perpendicular (see formula 4), or $a\,e = a\,f \times$ nat. cosine of $f\,a\,e$; $a\,f = 2'\ 9\frac{1}{8}'' = 2{\cdot}7604$ feet. The natural cosine of $f\,a\,e$ or 36° is 0·8·9017; by substitution of these values in the last equation, we have $e\,f = 2{\cdot}2332$ feet. Therefore $a\,h$ minus $a\,e$, or 2·7604 feet minus 2·2332 feet, is equal to 0·5272 ft. $= 6\frac{3}{4}$ inches or the horizontal upper joint of A and E. Q. E. D.

To obtain the horizontal joint of stones B and D, we perform a similar calculation with the triangle $a\,k\,m$. The central angle is 72°, obtained by adding to its own the preceding chord angle, the dotted lines being placed in the opposite quadrant, so as to avoid confusion of lines. Having obtained $a\,k$ (by formula 4), which by the way is just one half of the horizontal joint of the keystone, C, we subtract $a\,k$ from $a\,e$, and obtain the horizontal joints of B and D. Q. E. D.

3d. To determine the vertical joint of A, we resort

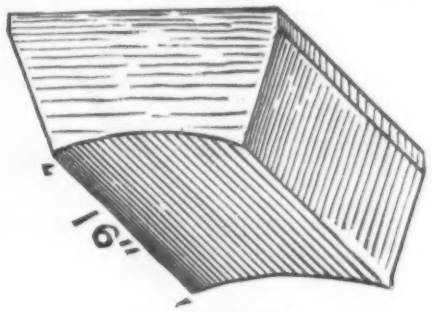


FIG. 7.

to the triangle $a\,e\,f$. This joint or $f\,e$ is equal to $g\,h$, and is determined by the formula (1) that the perpendicular of a right-angled triangle is equal to the hypotenuse into the sine of the angle at the base opposite the required side. Therefore, $f\,e = f\,a \times$ nat. sine of 36°, as we have seen before; $f\,a\,e = 36°$, $f\,a = 2{\cdot}7604$ ft.; natural sine of 36° is 0·587785; by substitution and multiplication we have $f\,a = 1{\cdot}6225$ ft. $= 1$ ft. $7\frac{1}{16}$ in. Q. E. D.

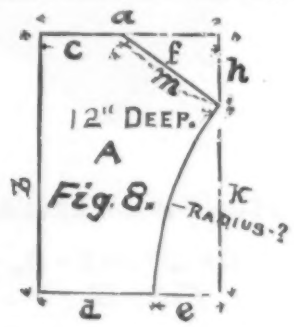
To determine the vertical joint of B and D, we solve in the same way the triangle $a\,k\,m$ with respect to the side $k\,m$; by formula 1, $k\,m = a\,m$ multiplied by natural sine of 72°, equal to 2·6252 ft.; if from $k\,m$ or 2·6252 ft. we subtract $f\,e$ or 1·6225 ft., we will have 1·0027 ft. $= 1$ ft. and $\frac{1}{32}$ in. or the vertical joint of B or D. Q. E. D.

Knowing the distance $k\,m$ and the distance $a\,n$ (2·7604 ft.), in order to find the vertical joints of the keystone, it is only necessary to subtract $k\,m$ from $a\,n$, and we have 0·1352 ft. or $1\frac{5}{8}$ in. Q. E. D.

As the corresponding vertical and horizontal joints of stones D and E are respectively equal to those of B and A, it is unnecessary to solve them separately, and all the facial joints therefore are calculated.

As a check upon the calculation, it is wise to reverse the operation, that is, assume what was just proved to calculate what was first assumed. Thus, assume the vertical joints and chords, and calculate the radial joints and horizontals. By this proposition, in the triangle $a\,e\,f$, we know the angle at the center to be 36°, also the vertical distance $f\,e$ or 1·6225 ft., to find the side $a\,f$. Now $a\,f = f\,e$ divided by the natural sine of $f\,a\,e$, $1{\cdot}6225 \div 0{\cdot}587785 = 2{\cdot}7604$; but $b\,a$ or the radius is = to $2'\ 0''$, $b\,f = a\,f$ minus $a\,b$, therefore $b\,f = 0{\cdot}7604$ ft., or $9\frac{1}{8}$ in.

In Fig. 14 we have a plan of a set of horizontal voussoirs such as are seen in the water towers of gate houses. The set shown here are designed for the intersection of a horizontal outlet to the tower; the cross section of the outlet at the point of intersection is rectangular, therefore the upper surfaces and bed joints are parallel to the horizontal course joints. The rising waters from



the tower pour over the edges of the intersection into the chamber beyond, not shown in the drawing. Stones Nos. 4 and 10 are the foundations of a pier located in the center of the outlet to divide the inflowing water into two currents. The unfinished part of the circle is the brick wall of the tower. In the calculations necessary to obtain the dimensions of the voussoirs, the procedure was as follows:

The foundation of the central pier in the water chamber beyond the well curb was to be 11 feet 10 inches long, point to point, and to be three feet wide. This foundation was divided into two stones, Nos. 4 and 10. In showing the method pursued to calculate the dimensions of stones Nos. 8 and 12, it will be shown how the center of the tower was located with reference to the facades of the building. Conceiving the center of the tower to be properly located, we assume that the diameter of the circle continued passes through the center line of the pier, as indicated in the drawing. The arc intercepted by the sides of the pier is therefore measured by a chord of three feet. This stone becomes the keystone of the arch. From each side of the keystone two chords of three feet each are laid off by scale, and the remaining chords, which belong to stones Nos. 1 and 7, are calculated. They are equal to each other.

To ascertain the chords to these stones, we must find the angles at the center subtended by them. We have all the premises necessary to do this. If from the angle subtended by the diameter, which is 180°, we subtract the sum of the angles at the center of all the known chords, we will obtain the sum of the angles at the center subtended by the two equal unknown chords.

Take any one of the known chords, as the keystone, for instance. The radius drawn perpendicular to the chord will bisect it and the angle it subtends. Conceive the bisection to have taken place by a radial line drawn as indicated, connect the intersections of the chord and the arc with the center of the circle, also the point of bisection, and we have a right-angled triangle ($a\,b\,c$) whose perpendicular ($c\,b$) we know, by reason of bisection, to be 1 ft. 6 in., and its hypotenuse ($a\,c$), or radius of the circle, to be, by construction, 6 ft. 6 in.

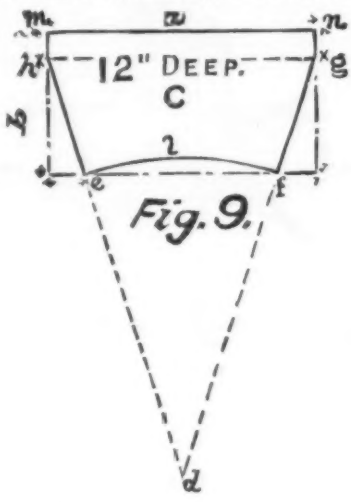


Fig. 9.

Solving this triangle with respect to the angle at the center ($c\,a\,b$), which is opposite the perpendicular, this angle is one half the total angle subtended by the chord, and we have from the formula: The natural sine of the angle opposite the perpendicular is equal to the perpendicular divided by the hypotenuse. (See last part of formula 1.) The perpendicular is 1·5 ft.; the hypotenuse is 6·5 ft. Substituting these values in the formula, and performing the indicated operations, we find the natural sine to be 0·230769. By referring to a table of natural sines, the corresponding angle is found to be 13° 20′ 32·44″. Doubling this angle, and discarding the decimal part of the seconds by adding one to the whole number, we have 26° 41′ 5″ as the angular measure of the chord of the keystone. We multiply this result by 5, as there are five equal chords, each subtended by this angle, to give us the sum of the angles at the center subtended by the sum of the five equal chords; thus $(26° 41' 5'') \times 5 = 133° 25' 25''$. Subtracting this sum from the total angular measure of the semicircle, or 180°, we have 46° 34′ 35″ as the angular measure of the sum of the two unknown chords; but these two chords are equal, therefore one of them is measured by one-half this angle, or 23° 17′ 17·5″. To find the chord corresponding to this angle, we reverse the process of finding the angle from the chord. By bisecting the angle at the center, and forming a right-angled triangle, one side of which is one-half the unknown chord, we have a triangle whose hypotenuse and the angle opposite the half chord (11° 38′ 38·75″) we know, to find one-half of the chord which is the perpendicular of the triangle. The formula used is: The perpendicular of a right-angled triangle is equal to the sine of the angle at the base opposite the perpendicular multiplied by the hypotenuse. (See formula 1.) The natural sine of 11° 38′ 38·75″ is 0·2018318. The hypotenuse is 6·5 ft. By substitution of these values, we obtain 1·3119 ft. for the half chord; doubling it, we obtain 2·6238 ft., or $2'\ 7\frac{3}{4}''$ for the chords of stones Nos. 1 and 7. Q. E. D.

Now that we know all the chords, and all of the radial joints are known by construction, we take the same course of procedure as indicated in the calculations of the vertical voussoirs, to obtain the vertical and horizontal joints to stones Nos. 1, 2, 6, and 7.

To find the vertical joints of Nos. 3 and 5, we subtract, found by previous calculation, but results only given here, the sum of the vertical joints of stones Nos. 2 and 3 from the distance of the horizontal joint of 3 or 5 to the springing line of the arch. To find the distance $a\,e$, we have $a\,e = a\,b + b\,e$; $a\,b$ can be found from the right-angled triangle $a\,b\,c$; $a\,b = a\,c \times$ cosine of $b\,a\,c$ (by formula 4), or $b\,a = \sqrt{a\,c - c\,b}$ (by formula 6); $b\,e$ we know to be $2'\ 0''$. The vertical joint of No. 3 or 5 is then found by subtracting from $a\,e$ the sum of the vertical joints of Nos. 2 and 1. Q. E. D.

To locate the center of the tower with reference to the main walls of the building, the following method was pursued:

Let $p\ p'$ be a portion of the face of the interior wall of the gate house chamber, and $p^2\ l$ a portion of a face

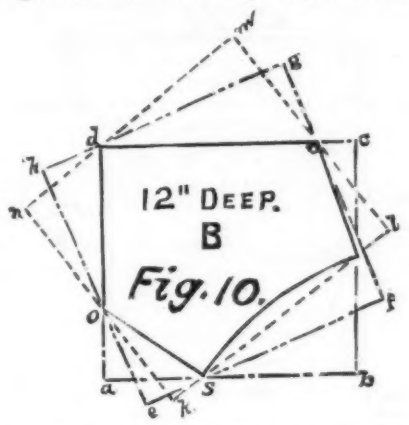


Fig. 10.

of an interior wall at right angles to $p\ p'$. The exact location of the pts. p or p^2 are unknown; at right angles to $p\ p'$ lay off $p'\ o' = 5'\ 9''$; at right angles to $p^2\ l$ lay off a line $l\ l^2$ equal to $6'\ 0''$; from the points o' and l^2 draw two lines $o'\ a$ and l^2 parallel respectively to $p\ p'$ and $p^2\ l$ and intersecting at the point a, which will be the center of the tower. With a as a center and a radius equal to $6'\ 6''$ describe a circle; the circle so described will be the horizontal projection of the circumference of the tower, that is, its interior circumference. Because the two lines which intersect at a are parallel to two lines which are at right angles to each other, it follows that the lines so drawn are also at right angles to each other, therefore the angle $o'\ a\ l^2$ is a right angle; bisect this right angle, and prolong the line of bisection through the point b. Through the points m and f draw lines parallel to $a\ b$; they will intersect the lines $p\ p'$ and $p^2\ l$ in the points p and p^2 and their distances from the line $m\,f$ are required, and all the dimensions of stones Nos. 8 and 12, except the distances marked $2'\ 0''$, which are given. The distance $a\ o$ is also required, for the distance of a given point within the building to a line passing through a at right angles to $p\ p'$ is 27 ft.; then, if $o\ a$ is known, the distance of p from the same given point can be determined.

The central line of the pier is made coincident with the line which bisects the right angle at a. The lines $m\ p$ and $f\ p^2$ are tangent respectively to the circle at the points m and f, therefore $m\ a\ b'$ is a right angle. The two triangles $a\ m\ n$ and $a\ b'\ n$ have their sides respectively parallel each to each; they are therefore

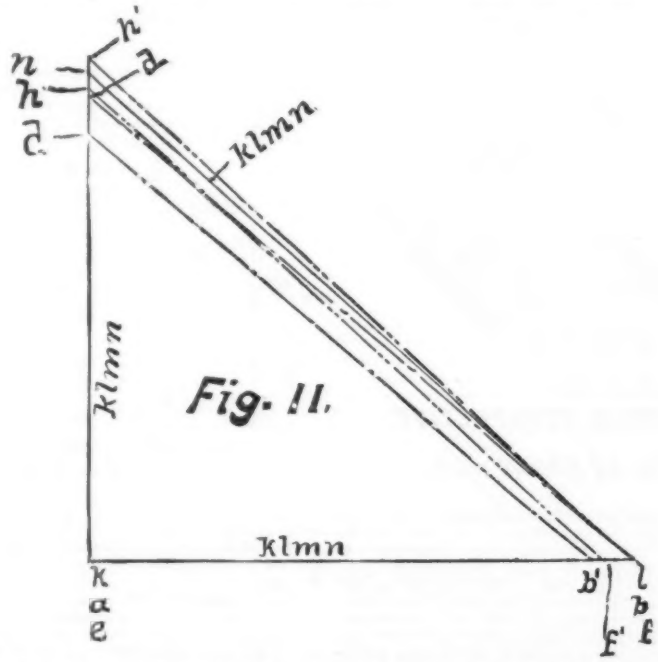


Fig. 11.

similar and equiangular, but the angle $n\ a\ b'$, by construction, is 45°, therefore $m\ a\ n$ is 45°, therefore $m\ n\ a$ is 45°. The triangle $a\ m\ n$ is therefore isosceles, and $m\ n = a\ m$.

We now have to find the relations existing between the two triangles $a\ m\ n$ and $n\ o\ p$, in order to determine the distances $a\ o$ and $m\ p$. The principles involved in determining these distances are precisely the same for determining the distances $f\ p^2$ and $a\ l^2$, but only the former will be worked out to show the method of reasoning involved.

We have proved that $a\ m\ b$ is a right-angled isosceles triangle, therefore $m\ n$ = to 6′ 6″, because $a\ m$ is 6′ 6″. Now $a\ n$ can be solved by two methods, either by the method of squares or by the formula for obtaining the hypotenuse when the perpendicular and the angle opposite is given. Using either method, the result is $9'\ 2\frac{5}{16}'' = a\ n$. All the elements of the triangle $a\ m\ n$ now are known.

Observing how the two triangles $a\ m\ n$ and $n\ o\ p$ are

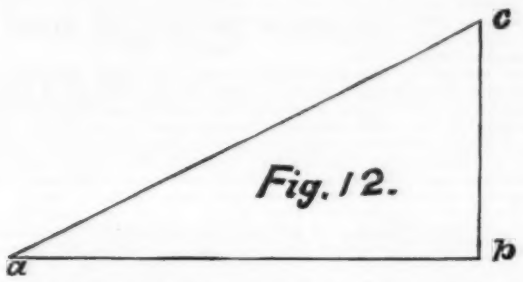


Fig. 12.

formed by the intersection of their own sides prolonged, we know from a principle of geometry that when two lines intersect each other, the angles opposite each other formed by such intersection are equal each to each; therefore the angle $o\ n\ p$ is equal to the angle $a\ n\ m$; but we have proved $a\ n\ m$ to be equal to 45°, therefore the angle $o\ n\ p$ is equal to 45°. Now the angle $n\ o\ p$ is a right angle by construction. The sum of the interior angles of every triangle is equal to 180°; if then $o\ n\ p + n\ p\ o + p\ o\ n = 180°$, and $p\ o\ n = 90°$, and $o\ n\ p = 45°$, we would have by substitution and transformation $o\ p\ n = 45°$; therefore the triangle $p\ o\ n$ is a right-angled isosceles triangle, and $n\ o = o\ p$; but $o\ p = 5'\ 9''$, therefore $o\ n = 5'\ 9''$. Now $o\ a = o\ n + n\ a$; $o\ n = 5'\ 9''$, $n\ a = 9'\ 2\frac{5}{16}''$, as we have seen; by substitution in the last equation, $o\ a = 14'\ 11\frac{5}{16}''$. Q. E. D.

In the triangle $n\ o\ p$ we have $o\ p$ and $o\ n$ given, and either angle opposite the right angle to be 45°; using either the method of squares or finding the hypotenuse when one side and the angle opposite is given as before, we find $n\ p$ = to 8·1317 feet; $m\ p = m\ n + n\ p$; $m\ n = 6'\ 6''$; $n\ p = 8'\ 1\frac{17}{32}''$; then will $m\ p = 14'\ 7\frac{11}{16}''$. Q. E. D.

The distance from m to s is found by adding together the vertical joints of Nos. 1 and 2 and the vertical joint of stone No. 13. Subtract this result from $m\ p$ and we have $s\ p$ or $2'\ 7\frac{11}{32}''$; the distance of s from s' is 2′ 0″, therefore $s'\ p$ is $7\frac{11}{32}''$. To show how $p\ r$ and the similar distance on stone 8 are obtained, an enlarged drawing has been made of stones 12 and 8; $r\ u$ is fixed at 2 ft., $t\ s$ is also 2 ft.; from p draw $p\ y$ parallel to $t\ s$ and at right angles to $s\ p$ and $t\ y$; prolong $p\ r$ to z; prolong $t\ y$ to z; the angle $z\ p\ y$ is equal to $o\ p\ n$, and is therefore 45°; $z\ y\ p$ is a right-angled isosceles triangle, therefore $z\ p\ y$ and $p\ z\ y$ are each 45°, and by the same method of squares or the value of the hypotenuse in terms of a side opposite and the included angle, as shown in the two previous demonstrations, we determine the hypotenuse $z\ p$ or $2'\ 9\frac{15}{16}''$; but $z\ r$ by construction is 2′ 0″, therefore $r\ p$ is $9\frac{15}{16}''$; but $y\ u = r\ p$ since $p\ y' = y\ y'$, and we thus obtain $t\ u$ directly by subtracting $y\ u$ or $9\frac{15}{16}$ from $y\ t$ or $2'\ 7\frac{11}{32}''$, and we obtain $1'\ 9\frac{11}{16}''$.

In calculating No. 8 the same principles are involved, but the line passing through the center of the tower at right angles to $a\ o$ is 6′ 0″ away from the facade of the building instead of 5′ 9″, and therefore the calculated distances will be different, as shown in the drawing.

MEASUREMENT.

In Figs. 8, 9, and 10, we show diagrams of stones A, B, and C, indicating by the arrows the dimensions required by the stonecutter. To arrive at the dimensions required, we scale off on the drawing, for instance, the distance a and b, and it is evident that these distances must be taken as two of the co-ordinates of the cube, and the depth which must be marked on the face of the stone will be the remaining co-ordinate. The two former distances a and b must be accurately calculated; we have shown how b was obtained. In the triangle bounded by the lines $m\ h\ f$, the angle included between f and m is 36°, because m is parallel to $a\ b$ and f to $a\ d$, therefore the included angles are equal each to each; and in this way, going from corner to corner, inspect the unknown parts and find what relation they bear to known parts, or resolve them into triangles, and solve them by some of the methods we have used, or others equally simple. Often the solution of one unknown part will, by simple subtraction or addition, furnish two or three required distances, not only to the stone from which they came, but also to the adjoining stone. Always on the lookout for symmetrical triangles to obtain an unknown part by simple proportion, or arranging constructed triangles with reference to obtaining the relation of alternate angles, is often of great service in economizing time and simplifying solutions.

In such a stone as B (see Fig. 10), whose outline we give surrounded by three different squares, viz., $a\ b\ c\ d$, $e\ f\ g\ h$, and $k\ l\ m\ n$, it is difficult sometimes without an elaborate calculation to tell how small a cube the voussoir can be cut out of. It is perhaps needless to say that for economy's sake the dimensions of the smallest cubical body out of which the given voussoir can be cut must be the one ordered from the quarryman. In order to avoid the tediousness of so much calculation, and at the same time to insure the ordering of the smallest cube possible, we proceed by the graphic method to compare the different dotted squares or rectangles to a system of rectangles having a common base. In the first place the rectangle $a\ b\ c\ d$ is laid out by inclosing the stone B, as indicated, and $e\ f\ g\ h$ is found by scaling across the apparently broadest space, as $o\ p$, and on it as a base construct a rectangle which shall just inclose the voussoir. Also scale the distance $s\ d$, and on it construct a rectangle which shall just inclose the voussoir. By scale ascertain the longest side, and let this be the base of the triangle of comparison, as $k\ l$ for example; in the diagram of comparison, Fig. 11, we have doubled all the components in order to render the lines less confusing. In practice, this diagram should be drawn three times greater than the scale of the drawing it pertains to.

From a proposition in geometry we have the principle that rectangles having equal bases are to each other as their altitudes; therefore, if we can reduce all of these rectangles to a set of rectangles having an equal base, we can compare their altitudes by scale, and at once determine by the shortest altitude which rectangle is the smallest without any calculation; having determined that, we can proceed to its calculation. Let $k\ l$ = the common base (Fig. 11) and $k\ n$ its altitude, then will the triangle $k\ l\ n$ be a triangle whose area is equal to one-half the rectangle $k\ l\ m\ n$. In the rectangle $e\ f\ g\ h$ lay off on $k\ l$, $e\ f = e\ f$, and on $k\ n$, $k\ h' = e\ h$; draw $f'\ h$ parallel to $l\ h'$, obtaining $k\ h$, which is the perpendicular or the altitude of the triangle $h\ k\ l$, which is equal to one-half the area of $e\ f\ g\ h$. Also make $a\ b' = a\ b$ of the rectangle, and $a\ d' = a\ d$ of the rectangle; draw $b'\ d$ parallel to $l\ n$, and we have $d\ k$, the new altitude; now $d\ k\ l$ = one-half the area of $a\ b\ c\ d$. As all of the triangles we have obtained have a common base $k\ l$, and different altitudes, we can consider their altitudes as their areas, and we discover that the triangle, and therefore the rectangle, and therefore the cube of which $k\ d = a\ d$ forms a part, is the smallest by comparison, therefore $a\ b\ c\ d$ is the surface of the smallest cubical body from which the stone B can be cut. It is now calculated in the usual way, and the others are erased from the drawing. This simple and ready graphic method we would most earnestly recommend as saving a vast amount of labor and time; and when care is taken in scaling,

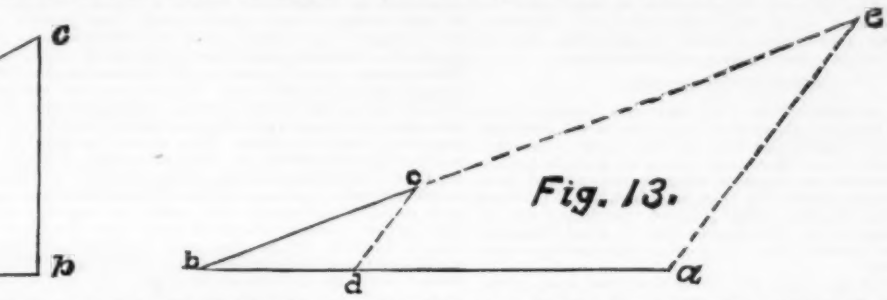


Fig. 13.

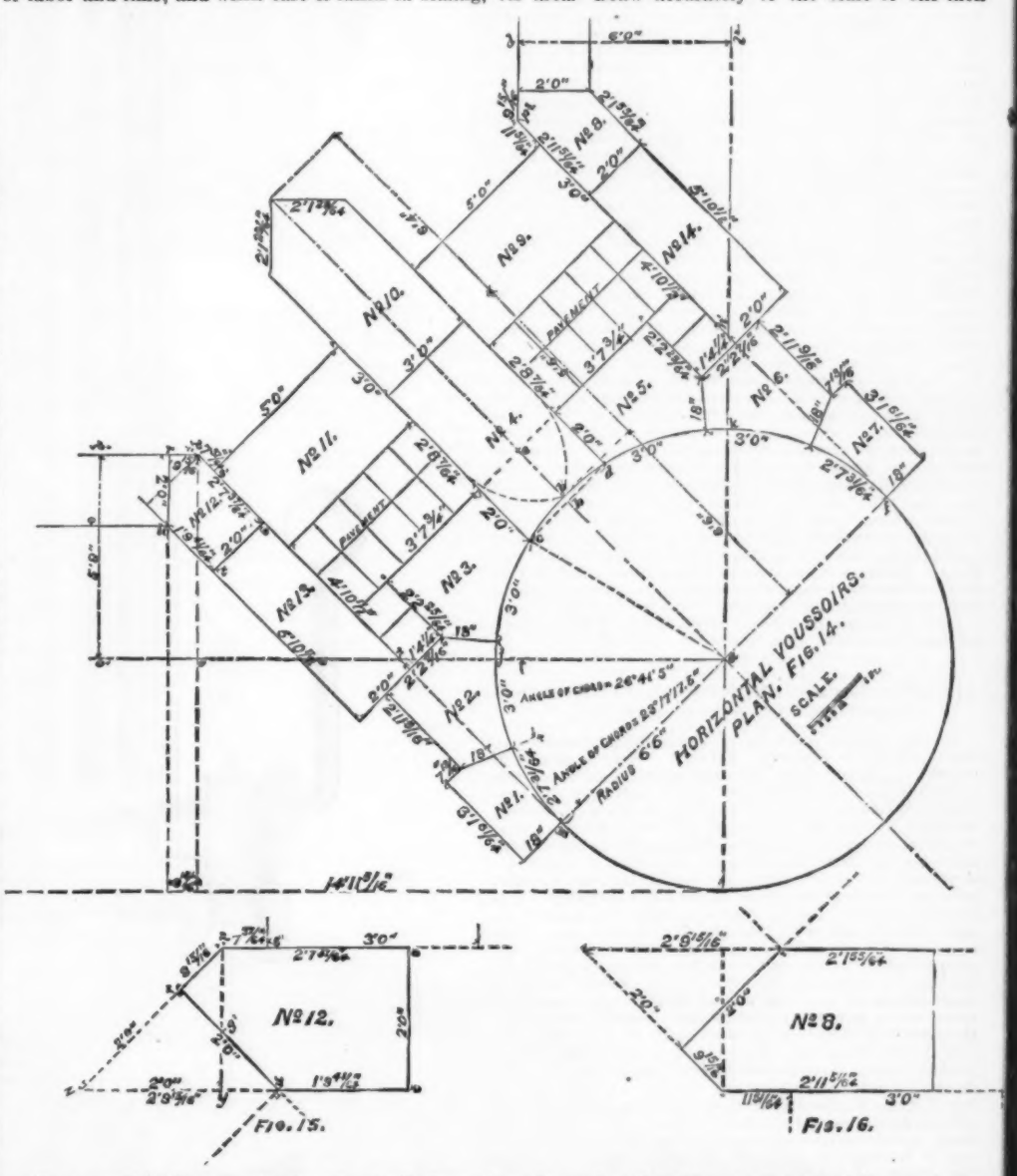
results are sufficiently accurate for all practical purposes.

Having obtained the area of the smallest rectangle which will contain the face of the voussoir, the cubical body required is obtained by multiplying the area so found by the depth assigned to the stone, or rather thickness. This will give the cubic feet in the stone; this is reduced to cubic yards either by some convenient conversion table, or in its absence by tedious calculation.

Where hammer dressed or surfacing work of any kind other than joint surface is required, the work is considered as extra to that given for quarry dimensions, and is paid for at so much per superficial foot. In such stones, only the surface of the finished face is paid for and not the full size of the corresponding quarry face, and therefore this area must be carefully calculated. This is done by resolving the face into triangles or parallelograms, by construction lines, and adding the sums of these areas in the usual way. For example, in Fig. 9, showing the keystone, C, the point indicated by d is the center of the circle of which $d\ e$ and $d\ f$ are radii, $e\ f$ is the chord of the arc of the keystone, and $h\ g$ is drawn parallel to the chord $h\ g$. By this construction we have two symmetrical triangles, $a\ f\ e$ and $a\ g\ h$; the solution of any distance, as $h\ g$, can be obtained, knowing $d\ f$, $f\ e$, and $d\ g$, as we have shown in a different calculation, and the area of the triangle $d\ g\ h$ can be obtained. If from this triangle we subtract the area of the sector $d\ f\ l\ e$, we will obtain the area of the figure $f\ l\ e\ h\ g$. To this add the area of the rectangle $g\ h\ m\ n$, and the area of C is obtained. In such or a similar way, the area of every dressed face can be obtained. To those who have no acquaintance with trigonometry or the higher mathematics, the method of graphical calculus offers the simplest solution of the difficult problems of areas and determination of lines, and at the same time great accuracy may be obtained in the results.

THE GRAPHICAL METHOD.

Let $a\ b\ c$ (Fig. 12) be any right-angled triangle, forming a part of some voussoir face. It is required to find its area. Draw accurately to the scale of one inch



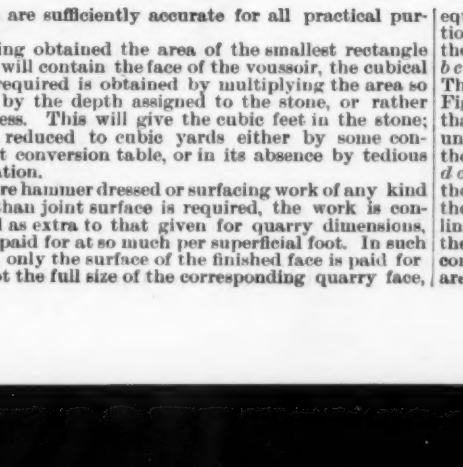


Fig. 15.

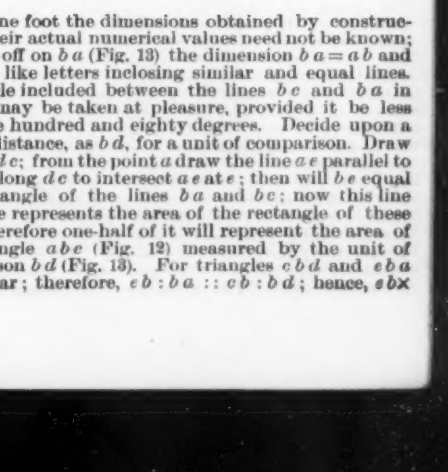


Fig. 16.

to the foot, $b\ c$ = to 6′ 6″, and $a\ b$ = to 9′ 2⅝″, and we have the data for finding the area. Now, as we have shown, $b\ c = b\ c$, like letters inclosing similar and equal lines. The angle included between the lines $b\ c$ and $b\ a$ in Fig. 13 may be taken at pleasure, provided it be less than one hundred and eighty degrees. Decide upon a unit of distance, as $b\ d$, for a unit of comparison. Draw the line $d\ c$; from the point a draw the line $a\ e$ parallel to $d\ c$; prolong $d\ c$ to intersect $a\ e$ at e; then will $b\ e$ equal the rectangle of the lines $b\ a$ and $b\ c$; now this line therefore represents the area of the rectangle of these lines, therefore one-half of it will represent the area of the triangle $a\ b\ c$ (Fig. 12) measured by the unit of comparison $b\ d$ (Fig. 13). For triangles $c\ b\ d$ and $e\ b\ a$ are similar; therefore, $e\ b : b\ a :: c\ b : b\ d$; hence, $e\ b \times$

$b\,d = a\,b \times b\,c$. If now we divide $e\,b \times b\,d$ by our unit $b\,d$, we will have $e\,b = a\,b \times b\,c$. Q. E. D.

From a well known principle of geometry we have the area of a triangle is equal to one-half of a rectangle having the same base and altitude. If the unit of comparison be taken as one inch on the same scale as the drawing, the length of the graphic line can at once be read off on the scale. By the aid of graphical calculus, the areas of any figures, the products of lines, the extraction of roots, the measure of angles by the lengths of their chords subtending them, and the solution of many problems, difficult analytically, can be rendered simple, time saving, and interesting.

SUMMARY OF METHODS.

When an outline of a set of voussoirs is given to an engineer or a draughtsman to calculate, it will be found expedient to determine first all the chords. In some cases where it is of great importance to work in the extrados joint of the voussoir, with the intersected horizontal course of the wall, pier, or abutment, it becomes necessary to fix the radial joints or vertical joints in such a manner that the chords exhibit great irregularity; even in such a case, it is good practice to begin with the chords.

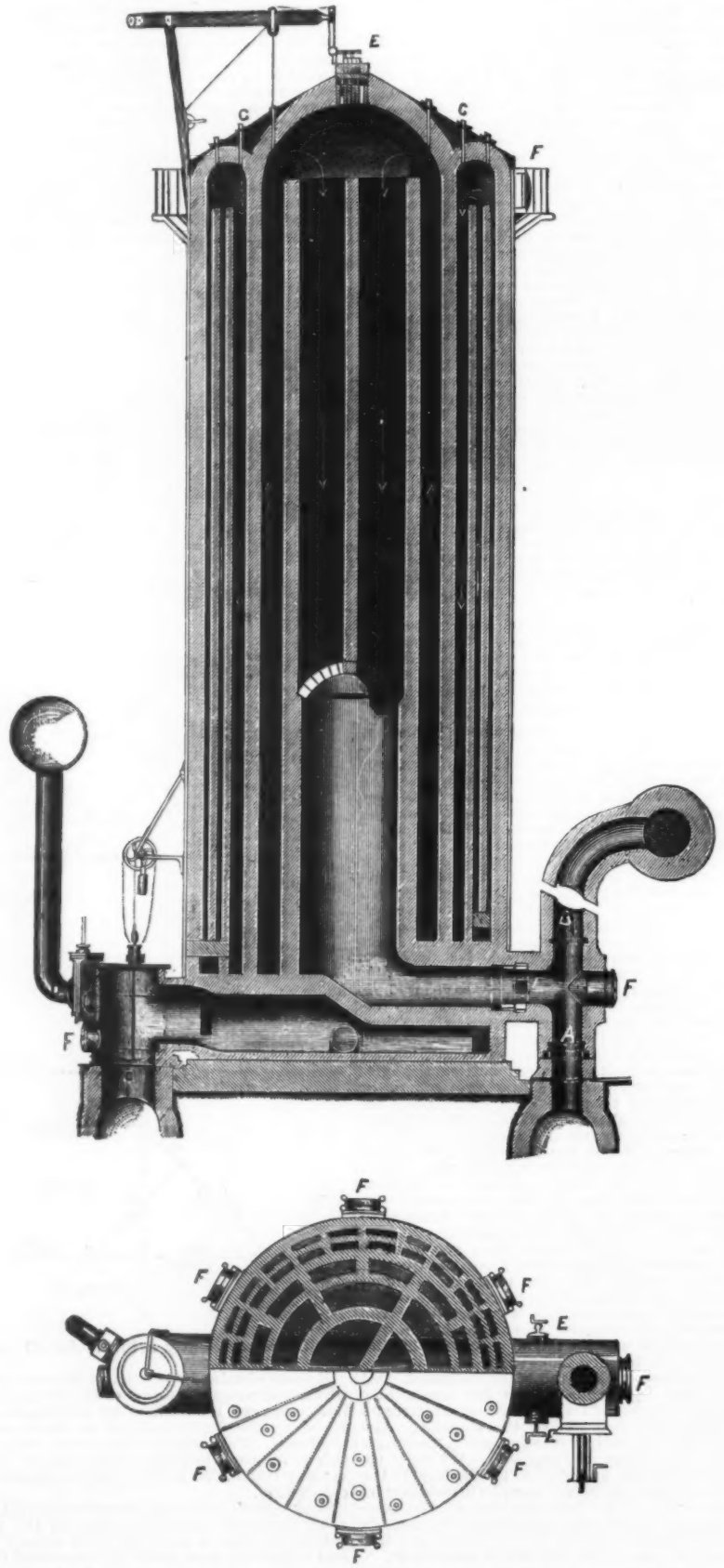
Beginners generally attack the skewback stone of the arch first, and work it out completely, and so on in order all around; but more rapid work may be done generally by starting with the first unknown radial joint, and so on until all the radial joints are determined. Then take up all the vertical joints, beginning always with the first arch stone on the left-hand side. Then take up the horizontal joints, beginning at the left hand as before. When the vertical joints are determined, add them up decimally and fractionally, to see how closely the total known vertical distance compares with the result; similarly proceed with the summation of the horizontal joints. Generally, the fractional results will check up admirably, but the decimal results will fall short of the true figure. A counter check on the verticals can sometimes be had, in sections or pairs, by referring to the established elevations of horizontal courses; when no error can be discovered by this method, the final resort is had to a summation of excesses over the fractional values, and also to a summation of deficiencies; the balance struck by combining the two results will generally indicate an error of not greater then one sixty-fourth of an inch. To the stone showing the greatest discrepancy add or subtract the balanced error, as the case may be, and the results may then be depended upon. In thus tracing an error to its source, the student or engineer will feel more confidence in the accuracy of his work. The writer would impress upon the minds of all who desire to attain the utmost accuracy in their work, this principle: Never use a deduced result as a factor of a formula for calculating an unknown quantity—if you can help it; or, in other words, always use the figures that have been arbitrarily fixed at the start so far as possible to obtain the unknown side or angle. The reason is obvious. If you use a quantity which is the result of your own work, and it contained an error, that error would vitiate all your subsequent work. Suppose you are obliged to use the value of a given angle to determine some unknown side; suppose this side is then found and you find it more convenient to use this calculated side than the given angle in subsequent calculations; the natural tendency would be of course to use that method which will entail the least work; but in voussoir work, at least, experience will teach you that the method which will produce the most accurate results is the shortest method that can be devised. Take no chances of error, for all fine stonework calculations are generally worked out by two or more engineers at the same time, and if the results agree to the one sixty-fourth of an inch, the results are accepted. Carelessness of method will cause more distrust of one's work than a few unimportant numerical errors now and then. Whatever course of procedure you mark out in the work before you, follow it faithfully, for systematic procedure in all mathematical work is conducive to accuracy in results, and in no department of work more so than in voussoir calculation.

RECENT IMPROVEMENTS IN THE MANUFACTURE OF RIFLE BARRELS.

AT the recent meeting of the British Association, Mr. Arthur Greenwood, M.I.C.E., read a paper in which he stated that the introduction of steel rifle barrels some five-and-twenty years ago, and the gradual substitution of that material in place of welded wrought iron for the barrels of military rifles and arms of precision, had caused great changes in the machinery employed in their manufacture, and that a short description of such machinery could not be without interest in this great center of firearms industry. The barrels generally used for military rifles are made from mild steel rolled solid and the bore drilled out. This is necessarily a costly operation, and numerous attempts have been made at hollow rolling to avoid the expense of drilling. To effect this, barrel blanks or short pieces of steel have been drilled and afterward rolled on a mandrel to the desired lengths, and many thousands of barrels have been made on this plan. Of late years, however, the military authorities have insisted upon barrels being first rolled solid to the desired form and afterward drilled. At the Royal Small Arms Factory at Enfield a system of continuous rolling has been established, whereby the barrel has been rolled to its exterior form at one heat by passing it through a number of rolls placed one after the other. A barrel is thus rolled in about half a minute, which gives a great saving in time and cost of production, and the material is not harmed by reheating, as was often the case in the old system. When the rolled barrel has become cold, it is passed through a straightening machine, which takes out any short bends or kinks, and is thus made ready for drilling. Before going to the drilling machine, the barrel goes to a machine which forces or squares up the ends, and prepares a bearing at each end of it upon which it will rotate in the barrel-drilling machine. The barrel-drilling machine has been specially designed to drill a perfectly true hole through the barrel, and to avoid straightening by means of external hammering, which is extremely harmful to the barrel. Three barrels are operated upon at one time, and drilled simultaneously from both ends. The machine is horizontal, and the barrels rotate at from 700 to 900 revolutions per minute. The drills are of the form technically known as D or half round bits, and are fed up automatically, a heavy pressure of water being injected up the barrels, so as effectually to wash out the cuttings or swath, and thus insure the free action of the drills. About thirty barrels can thus be drilled in the working day by one attendant. This system reduces the cost of drilling to little more than an eighth of the former system employed at Enfield, the results as regards excellence of work being at least equally good. The barrels are then rough or quick bored, and then filled up, or fine bored, in horizontal machines by means of square bits. This system is well known, and has undergone little change during recent years. The exterior of the barrel is next turned, and ingenious arrangements have been made to secure the exterior being turned true or concentric with the bore. The system at present at work at Enfield effects this with admirable precision, a bush, or temporary bearing, being fixed to the barrel concentric with the bore. This bush revolves in a stay, and steadies the barrel until spots or bearings have been turned upon it, and from these spots the barrel is rough turned. It is afterward finished in a copy-turning lathe to the desired form, the result being a barrel with its exterior practically concentric with its bore, which is absolutely necessary for good shooting. In rifling, little change has been made in the system, but much attention has lately been given to reduce the cost of this delicate operation. The automatic rifling machine is coming generally into use, and now, with the automatic machine which rotates the barrel after each cut, gives the feed to the cutters, and finally rings a bell to summon the attendant when the barrel is completed. One man can attend to a number of machines, producing as many as sixty or seventy barrels per working day, whereas on the old system one attendant was required for each machine, producing on an average twenty barrels per day.

MASSICK & CROOKES' HOT BLAST FIRE BRICK STOVE.

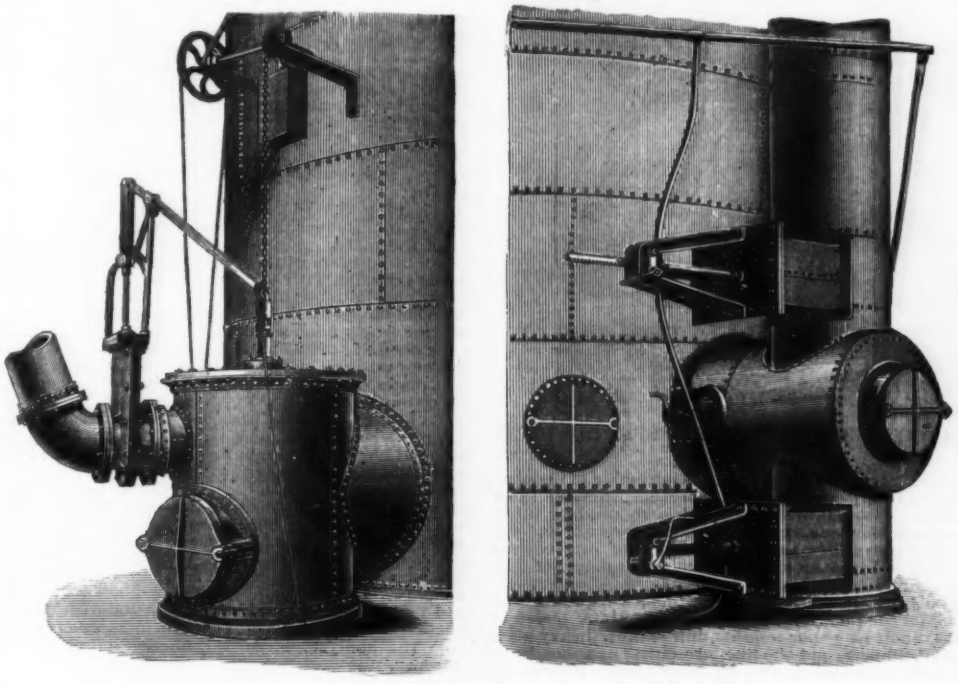
WE give views of this hot blast fire brick stove, as erected at the works of the Askham & Mousell Iron Company, Limited, at Askham-in-Furness. This stove is the outcome of many years of experience obtained in the management and working of some of the largest of the Cumberland hematite iron works, and has been designed in order to avoid some of the many objections and difficulties that arise in the working of other patent hot blast stoves. It is, we know, the universal opinion of all ironmasters of any considerable experience that it is vain to look for much increase in the prices of pig iron again. There may be, and no doubt there will be, fluctuations from time to time; but the good old days when pigs sold at 140s. per ton have gone never to return, and it is useless to mourn after



MASSICK & CROOKES' HOT BLAST FIRE BRICK STOVE.

those days. Ten years ago, many furnaces were considered to have done exceedingly well when they could produce their 300 tons of pig iron per week, with a consumption of coke equal to 22 cwt. to 23 cwt. per ton of iron produced; but those times are altered now, and at many of the best works, where appliances such as we now illustrate are fixed, a production of over 700 tons of pig iron is effected, the expenditure for labor remaining the same, but the consumption of coke being reduced from 23 cwt. to 18 cwt. per ton of pig iron. A no less important item, viz., the wear and tear of plant, has been reduced to less than one-half of what it formerly was.

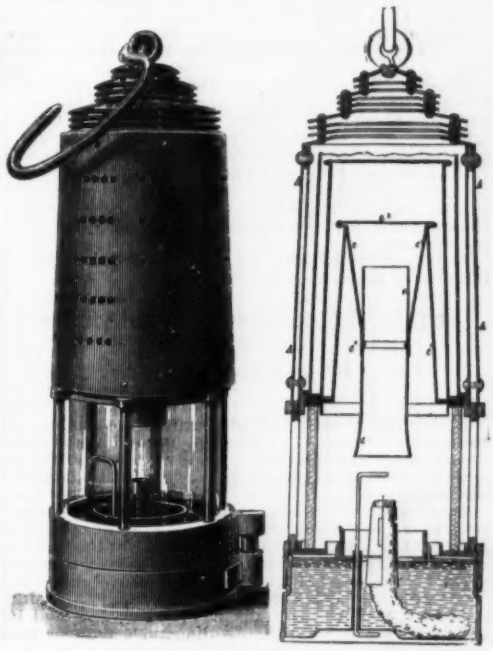
The view shown below has been taken from the works of the Askham & Mousell Iron Company, where the number of furnaces are four, which are fitted with fifteen of the patent stoves. These stoves have the merit of extreme simplicity, nearly the whole of the brickwork being formed of ordinary shaped fire bricks, thus saving a considerable item, both in time and



money, when compared with those stoves where bricks and moulds of many and special sections are employed. The stoves are "regenerative," being built somewhat on the principle of Messrs. Siemens' well known furnaces, the details of which need not here be gone into. Another good feature in this stove is the largely increased area of the passages, which tends in no small degree to augment the results of production at the furnaces. We have known of many instances where old plants with iron pipe stoves have required double the blowing cylinder capacity per ton of iron produced compared with the modern plant now working with these stoves. This, of course, is more than a small matter, when it is considered that from 160,000 to 200,000 cub. ft. of air are required for the production of one ton of pig iron. The arrangement of central heat is also a most admirable feature of this stove, for it answers the double purpose of preserving the outer or iron casing and at the same time preventing the escape of any heat. During the working of these stoves, the waste gases from the top of the blast furnace first enter through the middle of the stove into a large combustion chamber, formed for its reception. It is there mixed with a sufficient quantity of atmospheric air, in order to insure perfect combustion. Thence it travels down a second annular chamber, up and through a third, and finally down a fourth or outer chamber, by which time the gases have parted with the greater portion of their heat to the surrounding brickwork. After this it eventually descends under the center of the stove, thence to the chimney. When the stove is sufficiently heated, the gas and chimney valves are closed, and the valve, D, opened to admit cold air from the blowing engines. This air is admitted as near the outlet of the gas as possible, and takes a reverse direction to the gas. Entering under the stove, it passes up the outer walls, down the third chamber, up the second chamber, and thence it passes into the central combustion chamber, where it gathers its final heat, and thence carries it into the blast furnace. These stoves are made specially strong and capacious where the greatest heat is found, viz., in and around the center; but as the outer chambers are approached, the brickwork is made thinner, and the annular spaces are nearer together. By this means the greatest heating surface is obtained consistent with durability. Five years' experience of these stoves at the Askham & Mousell Company's works has so thoroughly demonstrated their great economy and satisfactory working to the directors of this company, that they have ordered a further extension of such plant to be put down, and this is now being proceeded with. It is, therefore, evident that in the Cumberland hematite district the pig iron manufacturers are fully alive to the importance of not waiting for better times; but they believe in putting their works into such a state of efficiency that they may command a profit even at present prices. We understand that the patentees have over thirty-eight of these patent stoves at work in this country and on the Continent, all of which are giving great satisfaction.—*Industries.*

THE "MORGAN" MINER'S SAFETY LAMP.

THE inventor claims for it some very special features, especially that when subjected to tests far more severe than are necessary to explode any of the other lamps in common use, the "Morgan" lamp will not pass the flame. It is claimed that the illuminating power of the lamp is higher than that of any other, and that the miner using it does not easily lose his light. These features are all the more striking as existing side by side in the same lamp; for the Royal Commissioners on Accidents in Mines, from their investigations of the many lamps submitted for their inspection, arrived at the opinion, says *Engineering*, that high illuminating power was not compatible with real safety; and, moreover, they were clearly of opinion that a lamp which was extinguished by the presence of firedamp was preferable to one which was not. Of course, if a miner had not confidence in his lamp, he would rather it were necessary he should grope his way out of the pit in the dark than that his lamp should continue burning in the presence of explosive mixtures of air and carbureted hydrogen. But if his lamp were able to resist extreme tests, we have no doubt he would prefer the benefit of its light while making away in haste from the quarter whence danger seemed to threaten. One of the greatest difficulties owners have had to encounter when substituting the Mueseler lamp, or the Marsant, for the Davy and the Clanny, is the vexation of the men in finding that the light of their new lamps so easily goes out. They say that with the Davy or the Clanny they are not under the necessity of going so frequently to the lamp station for a light, and never having seen an accident with the Davy or Clanny, they would rather go back to the old lamps. Then they strike against the new introduction, so that a really safe lamp, one which



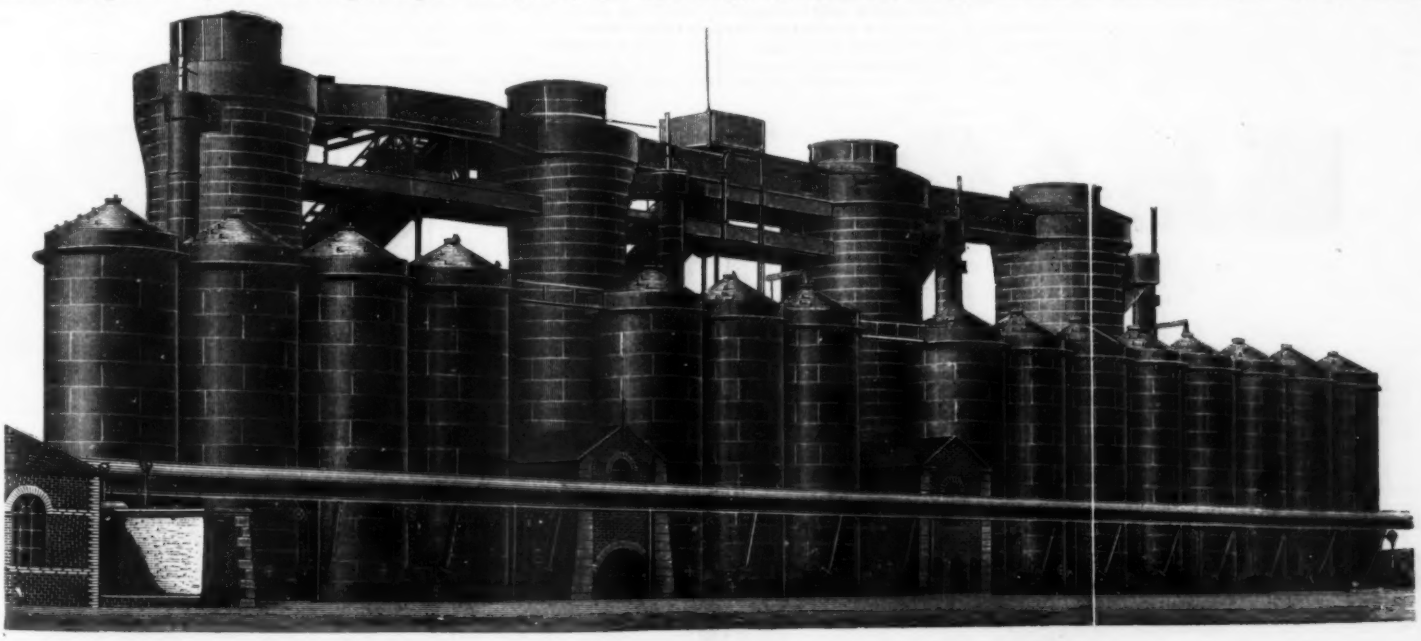
THE MORGAN SAFETY LAMP.

gives a good light and which will not easily be extinguished, if such a thing has been devised, must be hailed as a great boon alike by masters and men.

The experiments made to demonstrate the safety of the "Morgan" lamp appear to have satisfied many well known practical men, although Professor Lupton appears to have succeeded in making it fire; but as the usual tests can be applied wherever there is apparatus for the purpose, the claims of the lamp can easily be settled one way or the other.

The illustrations we publish above give a clear idea of the general form and the detailed construction of the lamp. There is a double shield, air being admitted through circular holes punched in rows in the outer air plate envelope. Air is also admitted between the two shields, both at top and bottom. The inner shield has long holes punched in it horizontally, but in no case are these opposite in situation to the holes of the outer shield. The holes in the inner shield alternate with those in the outer one in such a way that any current of air passing the lamp in considerably broken in its force. Hence the difficulty of causing the flame to waver. No current can be made to impinge upon the outside at any angle which will create a direct draught through the lamp.

The internal portions of the lamp are so arranged



MASSICK & CROOKES' HOT BLAST FIRE BRICK STOVE.

with respect to the admission of air to support the combustion of the oil, and the carrying off of the products of such combustion, as to obtain the maximum of brilliancy. There is no mingling of the carbon dioxide which results from the combustion of the oil with the oxygen needed for the flame. A central tube, which carries off the whole of the products of combustion, is supported at a short distance above the flame by means of a wire gauze cone, truncated and inverted. The upper end of the cone is secured to the upper end of the inner gauze. Under this arrangement, the clearing of the gauzes from any accumulation which may occur is extremely simple. As a matter of experience, it is found that only the gauze caps need to be cleared, except at long intervals, the sides remaining unclogged. The double shield prevents in a large measure the accumulation of coal dust on the outside gauze.

The lamp is locked by an exceedingly neat contrivance shown in the illustration. A projection at the base of the lamp comes directly over a similar projection on the oil box. The lower projection is perforated vertically, and into the passage is simply pushed a leaden pin, which goes through the lower projection and partly into the upper one. Inside the passage of the lower projection is a spring which bites a groove in the pin and prevents the latter being drawn back.

When it is required to open the lamp, the pin is cut between the projections; the upper half then falls out, while the lower half is simply pushed up, when it, too, falls away. The operation of locking and unlocking the lamp is thus effected with great expedition; although the miner cannot tamper with the arrangement without certain detection. The same ease and simplicity characterize the method of removing any other portions of the lamp for examination, and the replacement of the parts.

The lamp was adjudicated the Gold Medal at the International Inventions Exhibition.

SIMPLE EXPERIMENTS IN POLARIZED LIGHT.

By GEO. M. HOPKINS.

I.

IT is ever a source of pleasure to the student of science to be able to explore an unfamiliar realm by means of commonplace and readily accessible things, which, if not already possessed, may be had almost for the asking.

There is scarcely a branch of scientific research more prolific in the development of expensive apparatus than that of light, yet there is nothing in the domain of physics capable of being better illustrated by apparatus of the most simple and inexpensive character. The subject of polarized light, as intricate and difficult as it may at first appear, may be illustrated by apparatus costing less than a dime, in a manner that can but excite the wonder and admiration of one inexperienced in this direction.

A small piece of window glass and a black covered book constitute the apparatus for beginning the study of this interesting subject, and with a glass bottle stopper, a glass paper weight, or a piece of mica, the effects of polarized light may at once be shown.

SIMPLE FORM OF NORREMBERG DOUBLER.

The book is placed horizontally near a source of light, such as a window or a lamp, so that a broad beam of light will fall obliquely on it, and upon the book is placed the object to be examined, which may be either of those named.

Now, by viewing the reflected image of the object in the piece of window glass, with the glass arranged at the proper angle, it is probable that colors will be seen in the object. If no colors appear, it is due to one of three causes: either the object is incapable of depolarizing the light polarized by reflection from the book cover, or it is too thick or too thin to produce interference phenomena, or the eye of the observer and the glass employed for the analyzer are not in a correct position relative to the object and the polarizer (the book cover).

The glass, if thoroughly annealed, will produce no effect on the polarized beam, but most thick pieces of glass, such as paper weights, ink stands, heavy glass bottle stoppers, and the like, are either unannealed or only partly annealed, and are thus under permanent strain, which is readily indicated by their action on polarized light. A plate of mica of suitable thickness exhibits bright colors when examined by polarized light, particularly when the plate is either bowed or inclined.

To render the polariscope thus described more efficient, a plate of glass may be placed on the book, when the superior reflecting surface will at once make itself manifest in the increased brightness of the colors and improved definition of the object. A still greater improvement may be made by blacking one side of each glass with asphaltum varnish or any other convenient black varnish or paint, using in the experiments the unblackened surfaces, as shown in Fig. 1.

The angle which the incident light beam should make with the polarizer or horizontal blackened plate is 35° 25', and the polarized beam should strike the analyzing plate at the same angle, to secure the maximum effects; but it is unnecessary to measure the angles, as they may be easily determined by the appearance of the object.

With the two plates of blackened glass much may be learned with regard to the properties of polarized light. Plates of mica of various thicknesses and forms, inclined at various angles, bowed and turned in their own planes, pieces of quartz, bodies of glass such as those already mentioned, and odd-shaped pieces of unannealed glass, such as may be picked up at glass works, are easily secured objects. Brazilian pebble spectacle lenses often show gorgeous colors when turned at different angles in the beam of polarized light.

The writer well remembers the smiles provoked among the storekeepers when he visited various stationery and fancy goods establishments, armed with a Nicol prism, in quest of objects for the polariscope. As one article after another was placed upon a dark covered book, and examined with the prism, the writer imagined he could vaguely hear such words as Ward's Island, asylum, etc., and at this stage, as a matter of policy, the purchases were generally made, for sanity was at once established when the dealer examined the articles for himself with the Nicol, and the price of certain glass objects immediately advanced.

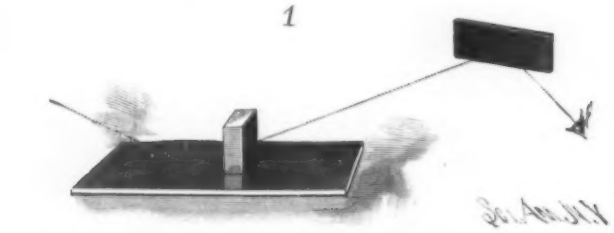
Seeing their own wares by polarized light, for some of the dealers, at least, was a new experience.

The best position for the polarizing plate is near a window, with the broad light of the clear sky shining upon it.

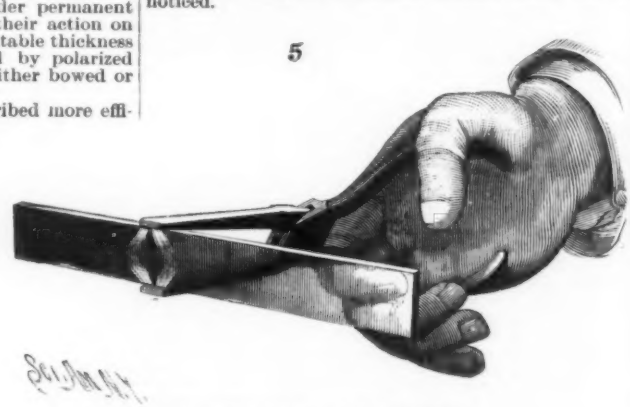
By turning the analyzing plate on the axis of the light beam, some curious effects may be observed. When the plates are at right angles with each other, the polarized beam will be nearly quenched,* and when they are parallel with each other, the reflection of the sky will be quite bright.

The employment of a blackened glass reflector for an analyzer is attended with some difficulty, on account of the necessity of changing the position of the eye for each new position of the analyzer. A bundle of six or eight plates of ordinary glass is more convenient, but not quite as efficient. These plates will be used as shown in Fig. 2, the light passing through them to the eye instead of being reflected. The plates may be turned at any angle without changing the position of the eye.

The most perfect analyzer, however, is the Nicol prism.† A very small one will answer perfectly for this class of experiments, and is not expensive. But to return to our experiments; when the analyzer and polarizer are crossed and the field is dark, if a few pieces of mica of various thicknesses and shapes are held between the analyzer and the black glass plate, and bowed and inclined at different angles, a great variety of tints will be observed, and if held in one position while the analyzer is turned, another effect will be noticed.



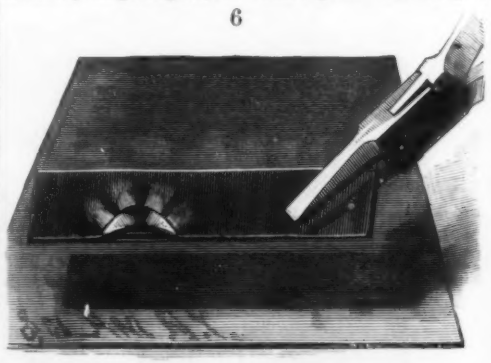
POLARIZATION BY REFLECTION FROM BLACKENED GLASS.



GLASS STRAINED BY PRESSURE.

ANALYZATION BY BUNDLE OF GLASS PLATES AND VERRE TREMPE.

Among the objects which may be examined in this way are the paper weights, stoppers, and other thick, partly annealed pieces of glass, a piece of glass held edgewise in a hand vise or pair of pliers, and put under compression, as shown in Fig. 5. A piece of glass held edgewise for a moment in a small gas or candle flame, and then placed in the polarized beam, shows the strain by a light figure, like that represented in Fig. 6,



GLASS STRAINED BY HEAT.

or it may assume other forms, according to circumstances. As the glass cools, the figure fades away.

Small glass squares and triangular and diamond-shaped plates, about three-quarter inch across, suspended by a fine wire in the flame of a Bunsen burner or alcohol lamp until their corners begin to fuse, and

* With black glass reflectors employed as polarizer and analyzer, the extinction of the light is not quite complete, even when they are arranged accurately at the polarizing angle. See paper on "Polarized Light," in SUPPLEMENT 538.

† For description see SUPPLEMENT 538.

then cooled in air, become permanently strained, and exhibit symmetrical figures formed of dark and light spaces, but show little color on account of their thinness. By superposing several such plates, color effects may be seen.

The beautiful *verre trempe*, or strained glass blocks, a few examples of which are represented at *a*, *b*, *c*, *d*, in Fig. 2, are similar in character to what has just been described. They vary in thickness from one fourth inch to one-half inch, and even thicker. They are expensive objects, but exceedingly beautiful and interesting.

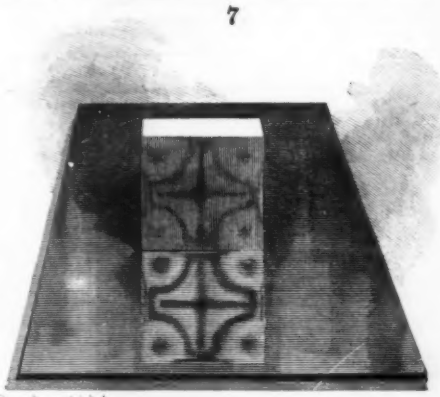
In Fig. 3 is shown a method of polarizing and analyzing with a single bundle of plates. It is, in principle, a Norremberg doubler. The light strikes the under surface bundle of plates at the polarizing angle, and is reflected downward in a polarized state, passing through the object which rests upon the horizontal silvered mirror. It is then reflected back through the object, and passes through the bundle of plates to the eye of the observer; the plates, as before stated, serving to analyze the polarized beam.

A Norremberg doubler, which answers a good purpose, may be made by leaning a clear plate of glass upon the edge of a book, over a piece of ordinary looking glass, and employing a bundle of glass plates as an analyzer, as shown in Fig. 4. Here the polarization is effected by the single plate of glass, and the analyzation by the bundle of plates held in the fingers. Equipped with this instrument, the student of polarized light may proceed a long way with his investigations.

In this instrument the objects to be examined are laid upon the horizontal mirror, and the inclined plate is arranged with reference to the light so that it will reflect the broad light of the sky downward. The position of the single plate and bundle of plates may be varied to secure the best effects.

In Fig. 7 is shown an arrangement by which the

7



DOUBLE POLARIZATION WITH SINGLE GLASS PLATE.

object and the blackened glass both act simultaneously as polarizer and analyzer. By placing a specimen of *verre trempe* edgewise on the blackened glass, as shown in the engraving, the light, striking the strained glass at about the polarizing angle, is reflected from the back surface of the glass and partly polarized. The beam thus polarized is reflected downward obliquely, and at the same time depolarized by the strained body of the glass; it is reflected upward to the eye and analyzed by the blackened glass mirror, thus producing an image which is apparently below the surface of the mirror. The image seen in the *verre trempe* itself is produced by the reverse of what has just been described. The light is polarized and reflected by the black glass mirror, and passes through to the back surface of the *verre trempe*, which reflects it back through the body of the glass; the glass then acts as both object and analyzer.

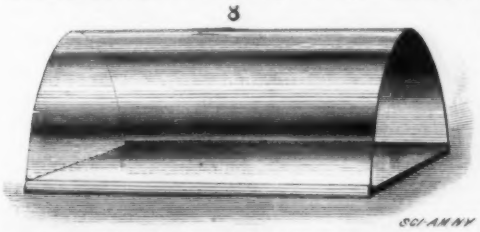
When the polarizer, analyzer, and object are each movable, different effects will be produced by rotating any of them. As a means of exhibiting complementary colors, nothing can excel the polariscope, as the colors produced in the successive changes resulting from turning the analyzer or polarizer are exactly complementary to each other. Little has been said about theories. The writer's object is to interest the reader in polarized light sufficiently to induce him to study the literature of the subject.

II.

Scientific experimentation, though practiced merely as a pastime, can but elevate the thoughts and bring the mind into new channels, thus promoting knowledge to some degree, even though the student proceed no further than the observation of effects. But once interested in effects, the inquisitive mind cannot rest satisfied without probing for causes.

So far as effects are concerned, the subject under consideration is everything that could be desired, and no great scientific knowledge or high manipulative skill is required to secure splendid results.

8



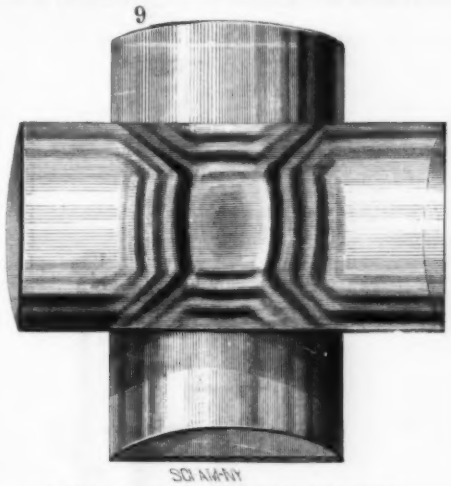
MICA SEMI-CYLINDER.

In the first part of this article, the writer mentions a few ways in which light could be polarized and analyzed, and gave hints as to some objects which might be viewed by polarized light.

A few simple objects easily prepared from mica are here shown. The material is of course procurable everywhere, and it requires little more than a glance at the engravings to enable any one to prepare the objects. Doubtless many other forms than those illustrated will suggest themselves to the student.

The simplest form is shown in Fig. 8. It consists of a thin plate of mica bowed into approximately semi-

9



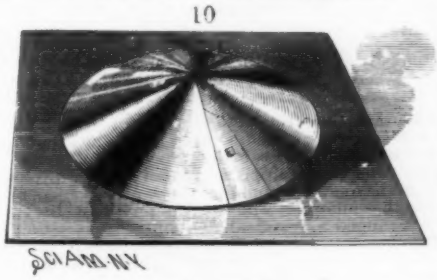
MICA SEMI-CYLINDERS CROSSED.

cylindrical form, and secured by its edges to a plate of glass by means of narrow strips of gummed paper. The size is immaterial; the glass plate may be 1½ in. wide by 3 in. long. This object exhibits fine bands of prismatic color when viewed in the polariscope. Two such semi-cylinders, when crossed, exhibit the intricate figure shown in Fig. 9, with all the splendid colors of the spectrum.

The object shown in Fig. 10 is formed of a disk of mica having a sector cut out and the radial edges overlapped, forming a low cone. The overlapping edges are best fastened together by small tin clips inserted in holes in the mica and bent downward on opposite sides. The clips are not noticeable, and are efficient in holding the edges together. Cement will not answer the purpose, as it adheres to the surface only, and it must be remembered that mica splits almost indefinitely.

The cone thus made has the appearance in the polariscope of a huge circular crystal of salicine. The colors of the cone may be heightened by mounting it on a sheet of mica, as shown in the engraving. The cone is first placed in the polariscope, with the polarizer and analyzer crossed, and turned until it appears brightest, when the lower edge is marked. The mica sheet is then placed in a similar way in the polariscope, and turned and marked. The cone is then cemented by its edges to the sheet, the marked edges of both members being arranged in the same direction.

10

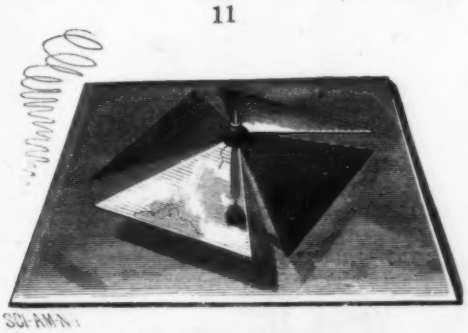


MICA CONE.

The Maltese cross shown in Fig. 11 is revoluble. The first step toward the preparation of this object is to secure a pin head downward on a square of glass with sealing wax or other cement. A small paper tube which will fit the pin loosely is then made, and a little head of sealing wax is formed around the tube near one end. A piece of mica is selected which exhibits fine colors in the polariscope, and four equilateral triangles are cut from it, either with their corresponding sides cut upon the same base line, or with one side of each cut from one side of a square, or they may be cut and mounted haphazard.

To the apex of the angle designed for attachment to

11



MALTESE CROSS.

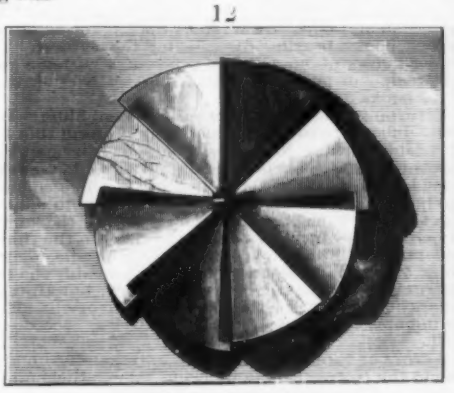
the paper tube a small drop of sealing wax is applied, and with the tube on the pin the first triangle is attached by holding it in the required position by means of a pair of tweezers, and then fusing the wax on the mica and that on the tube simultaneously by means of a small heated wire, such as a knitting needle.

The other members are placed and secured in a similar way, care being taken to arrange the triangles symmetrically, and at a slight angle with the plane of rotation of the object, as shown in the engraving.

The wheel shown in Fig. 12 and the star shown in Fig. 13 are prepared in a similar way. The sections of the wheel are cut from a circular piece of mica, and cemented in place on the paper tube after the fashion of a propeller wheel or wind wheel.

Each ray of the star is made of two scalene triangles of mica oppositely arranged with respect to each other, and inclined in opposite directions, the longer and shorter sides of adjacent triangles being fastened at the periphery of the star by a minute drop of sealing wax.

12

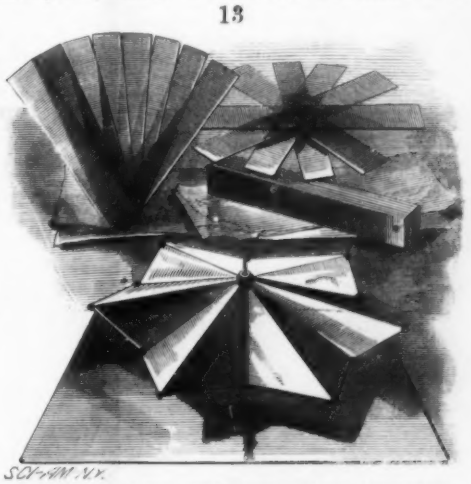


MICA WHEEL.

In Fig. 13, beside the star are shown two somewhat similar objects, formed of strips of mica, pivoted together on a small rivet, one object having the pivot in the center of the strips, the other having it at the end, giving the object an appearance similar to that of a folding fan.

Any of these objects may be viewed by means of the black glass polarizer in connection with either of the

13



STAR FAN AND CROSSED BARS OF MICA.

forms of analyzer already described or in the simple form of Norremberg doubler. These objects are also very satisfactory when projected on the screen.

III.

One of the simplest and best instruments for a certain class of investigations in polarized light is the Norremberg doubler, named after its inventor, and shown in a very simple form in the annexed engraving.

To one edge of a wooden base, 6 in. square and three-fourths of an inch thick, is secured a vertical standard, 1 in. square and about 15 in. high, and to the top of the standard is attached an arm extending over the center of the base, and apertured to receive the short tube containing the analyzing prism or bundle of glass plates. The tube may be made of paper, hard wood, or metal, and it should be fitted with a shoulder, so that it will turn readily in the aperture of the arm. To the standard below the arm is fitted a stage formed of a thin piece of wood centrally apertured and blackened.

The stage is notched to receive the standard, and is attached to a short vertical bar 1 in. wide. A clip of wood extending across the back of the bar, and two small clips secured to the sides of the short vertical bar, bear with sufficient friction on the standard to hold the stage in any desired position

About 6 in. above the base, a grooved wooden strip is pivoted to the standard, by means of a common wooden screw, passing loosely through the grooved strip and tightly through the standard. A wooden knob is turned on the end of the screw, and serves as a nut to bind the grooved strip in any desired position. The strip, screw, and knob are shown in detail in Fig. 15.

Into the groove of the strip is wedged or cemented a plate of glass, 4 by 9 in. A fine piece of ordinary window glass will answer, but plate glass is preferable.

Upon the base is laid a square of ordinary looking-glass, or, better, a piece of plate mirror.

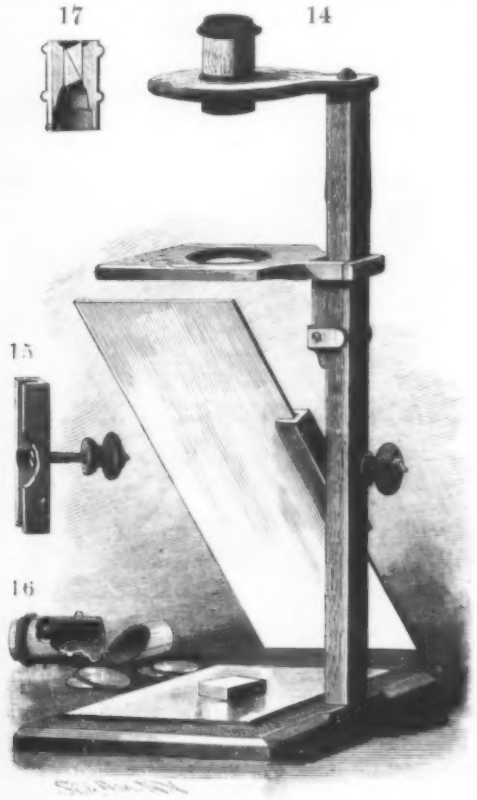
The tube shown in detail partly in section in Fig. 16 is provided with an inner tube of pasteboard or wood, divided obliquely at an angle of 35° 25' with the axis of the tube, and upon the oblique end of one-half of the tube are placed twelve or fifteen well cleaned elliptical microscope cover glasses, which are held in place by the other half of the divided tube. This bundle of glass plates, if of good quality and well cleaned, forms a very good analyzer; but instead of this, if it can be afforded, a small Nicol prism should be secured and mounted in a centrally apertured cork, the latter being inserted in the analyzer tube, as shown in Fig. 17.

The object to be examined may be laid either on the stage or on the mirror below. If viewed on the stage, the usual effects will be observed; but if laid on the mirror, it is traversed twice by the light, once by the incident beam and once by the reflected beam. This is particularly noticeable in thin films of mica and selenite, and it serves as an excellent means for selecting eighth and quarter wave plates, which are useful in the study of circular and elliptical polarization.

As stated in a former article, the writer intends to deal sparingly with the theoretical part of the subject, that having been treated extensively in many physical works and in books especially devoted to light and optics. "Ganot's Physics" is prominent among works of its class, and "Light," by Lewis Wright, and "Polarization of Light," by William Spottiswoode, are excellent books, bearing directly on the subject. The writer knows of no better means of securing a good knowledge of polarized light than by reading these three books.*

Returning to the matter of the thin films: It is quite difficult to produce a perfectly uniform thin film of selenite, owing to the brittleness of the material. For this reason, mica is generally used, as it possesses considerable flexibility and toughness. The common method of cleaving off thin films of mica is to split off a moderately thin plate and then separate the laminæ at one of the corners by bending it between the thumb and fingers. A medium sized sewing needle secured point outward in a slender handle is probably the best instrument for teasing the laminæ apart; but after the separation begins, the thin end of the ivory handle of an ink eraser seems to serve the purpose exceedingly well.

A score or so of plates are split, and examined one by one in the Norremberg doubler, by laying them on the mirror and turning them in their own planes, while the polarizer and analyzer are parallel. Should the plates exhibit any unevenness under the test, they should be



SIMPLE NORREMBERG DOUBLER.

at once rejected. Such as exhibit an even tint should be preserved carefully, and examined further to determine which, if any, possess the required qualities. Not every piece of mica will split evenly, therefore it may be necessary to make several trials before success is attained.

Should the film, when placed on the stage, exhibit a dull plum color, slightly inclined toward red, when the polarizer and analyzer are parallel, it produces a difference of phase of half a wave length, and is called a half wave film. As a matter of course, if two films of like thickness, superposed and arranged with their axes in the same direction, produce the same color under the same circumstances, they are one-fourth wave films; and if a pair of films exhibit the same color when similarly arranged on the mirror of the doubler, they may be regarded as eighth wave films, as the polarized beam passes twice through the film to produce the same tint. These films should be carefully mounted between glass plates, either dry or in benzole balsam, the latter being preferable.

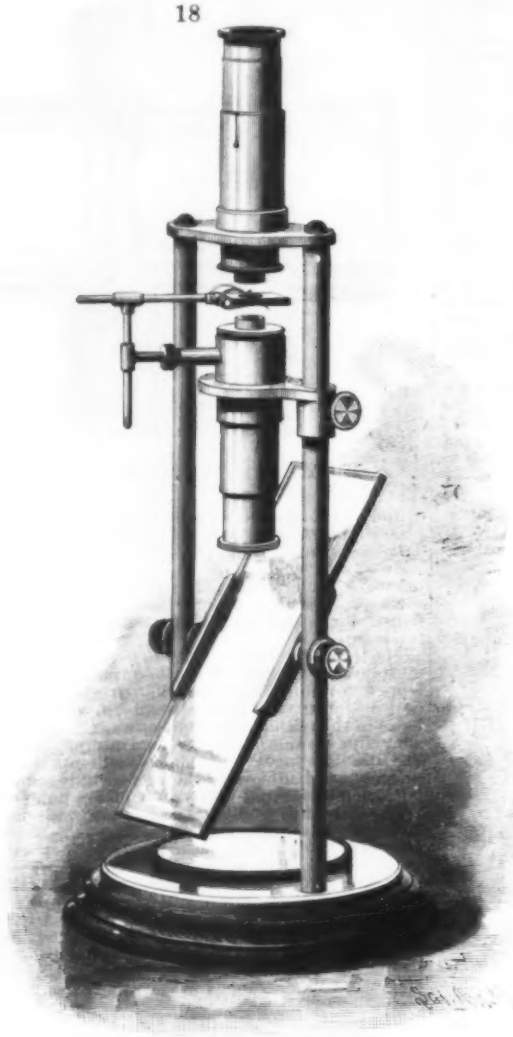
The practical application of the eighth and quarter wave films will be treated in a future paper. Beautiful and instructive designs made from thin films are described and illustrated in Wright's "Light," to which reference has been made.

IV.

The only simple device for exhibiting the rings and brushes of wide-angled crystals is the tourmaline tongs of the kind commonly employed by opticians for testing spectacle lenses; but the dark color of ordinary tourmaline renders a polariscope of this kind objectionable.

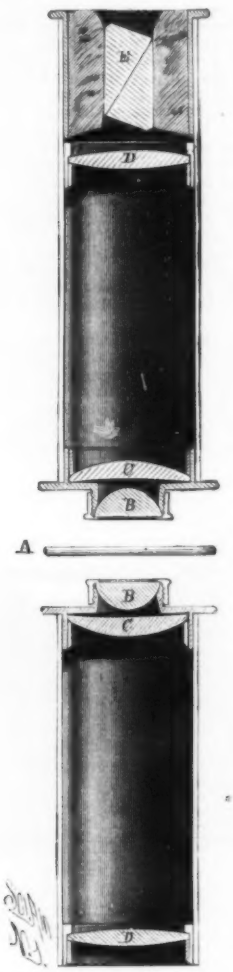
A system of lenses devised by Norremberg, and improved by Hoffman, is at present employed for observing the phenomena of wide-angled crystals; but it is a matter of some difficulty to secure exactly such lenses as are required for the apparatus as constructed by Hoffman. Very good results, however, may be obtained by the employment of lenses designed for other purposes. Reference is made to the hemispherical condensing lenses used by microscopists, and ordinary meniscus (periscopic) spectacle lenses. Six lenses in all are required. The converging and collecting systems are exactly alike, but they are oppositely arranged with respect to each other. In the present case the two systems are adapted to a Norremberg doubler

18



POLARISCOPE FOR EXHIBITING WIDE-ANGLED CRYSTALS.

19



LONGITUDINAL SECTION OF TUBES OF POLARISCOPE.

substantially like that described in a former part of this article, the main difference being that the instrument now illustrated is made principally of metal.

The tube of the upper system of lenses is prolonged upward beyond the upper lens to receive a Nicol prism, E, or other analyzer, which is mounted in a short inner tube arranged to revolve in the outer tube.

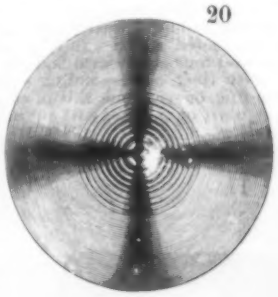
The lower system of lenses is contained by a tube fitted to the stage of the doubler. The arrangement of the lenses and analyzer is shown in Fig 19. The two systems of lenses being alike, a description of one will answer for both. The object, A, to be observed is held between the adjacent ends of the two tubes in the universal holder shown in Fig. 18.

The lens, B, next the object is nearly a hemisphere, about eleven-sixteenths in. in diameter and three-eighths in. focus. The second lens, C, a meniscus (periscopic) spectacle lens of 3 in. focus, is arranged with the concave face one-sixteenth in. from the convex side of the hemisphere. Beyond the 3 in. meniscus, 3½ in. distant, is placed a bi-convex spectacle lens, D, of 4 in. focus. The inner surfaces of the tubes are made dead black by the application of a varnish formed of lampblack, and alcohol in which only a trace of shellac has been dissolved.

The tubes may have any suitable diameter, and the proportions of the doubler may be about the same as indicated by Fig. 18, which is one-quarter actual size. The tubes and lenses shown in Fig. 19 are one-half size. The exact proportions, except as to the focal lengths and distances apart of the lenses, are immaterial. The lower system of lenses must produce a very convergent beam of light, while the upper system is arranged to collect the rays after they pass through the crystal, and bring them within the range of vision.

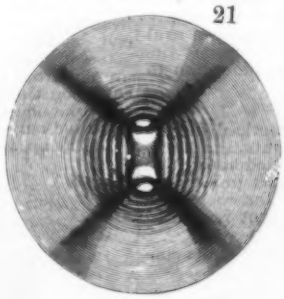
The angle between the optic axes in some crystals is so small as to permit of seeing them readily. Niter and carbonate of lead are examples of such crystals, but there are other crystals whose angle is so great as to render it exceedingly difficult to exhibit them, and in some crystals the angle is so wide as to render it im-

20



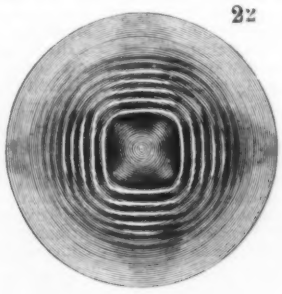
CALCITE.

21



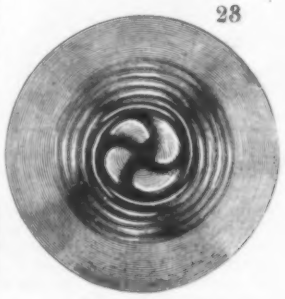
NITER.

22



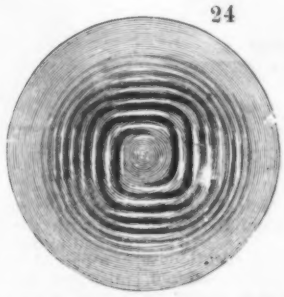
QUARTZ.

23



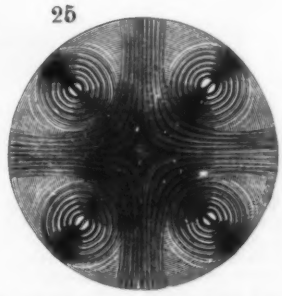
AIRY'S SPIRALS.

24



QUARTZ POLARIZED CIRCULARLY.

25



CALCITE HEMITROPE.

* These books may be had at this office at publishers' prices.—ED.

possible to see both axes at once. The only method of exhibiting them is by tilting the crystal first in one direction and then in the other, and viewing them separately.

Figs. 20 to 25 inclusive represent the figures shown by several crystals in the instrument illustrated. The drawings, having been made directly from the objects by the aid of the instrument, are correct in form and proportion, but the beautiful coloring is necessarily absent.

Fig. 20 shows the rings and brushes exhibited by calcite in a convergent beam of polarized light, with the polarizer and analyzer crossed. With the polarizer and analyzer parallel, the dark cross is replaced by a white one.

Niter is shown in Fig. 21 as it appears when the analyzer is crossed. With the analyzer parallel with the polarizing plate, the dark brushes are replaced by light ones. Turning the crystal in its own plane produces different effects.

In Fig. 22 is shown a figure produced by a slice of quartz cut at right angles to the axis of the crystal, and examined in the instrument with the analyzer arranged at an angle of 45° with the polarizer. Crystals of quartz vary in their effects on the polarized beam, some requiring the turning of the analyzer to the right, and others to the left to produce like results. For this reason the plates are called right or left handed, according to the direction in which the analyzer is required to be turned.

By superposing a right hand quartz on a left hand quartz, the beautiful spirals discovered by Airy, and named after their discoverer, may be exhibited. These spirals are shown in Fig. 23.

In Fig. 24 is shown the figure produced by the interposition of a quarter wave mica film between the polarizer and a plate of quartz viewed in the instrument. This altered appearance is due to circular polarization, a phenomenon readily understood on reading the literature of the subject, but requiring an explanation too elaborate for the space at command.

Calcite polarized circularly shows singularly broken up and disjointed rings, the brush-like cross being absent; and when analyzed circularly, or viewed through a quarter wave plate, as well as through the analyzer, the rings appear perfect, and there are no transverse markings.

Fig. 25 shows the intricate figure produced by a calcite hemitrope, or pair of crystals arranged at right angles with each other. Somewhat similar figures are produced by crossed plates of mica.

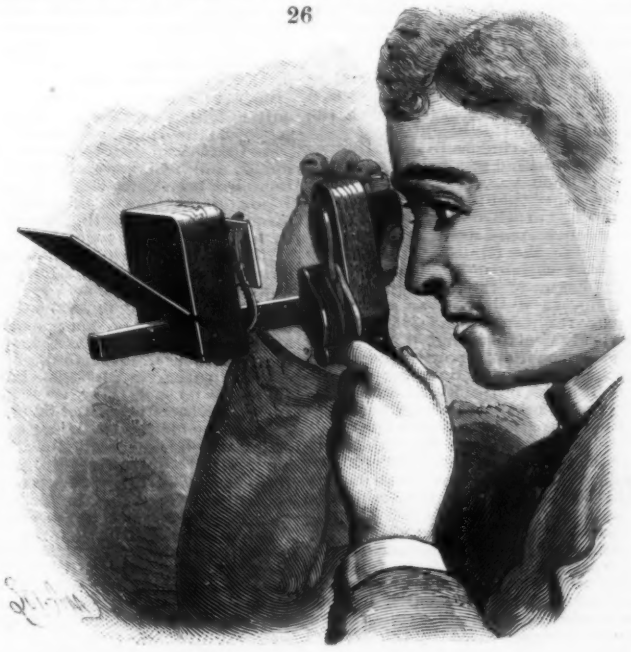
The following is a list of some additional objects which may be viewed in the instrument: Sulphate of nickel, sugar, aragonite, bichromate of potash, chrysoberyl chrysolite, topaz, anhydrite.

Instead of employing the Norremberg doubler for polarization, the lower tube may be prolonged, and a large Nicol prism inserted and arranged like the analyzer.

V.

The examination of microscopic crystals by the aid of the polariscope is an exceedingly interesting part of the study of polarized light. The indescribable play of colors, and the variety of exquisite forms of the smaller crystals, render this branch of the subject very fascinating. But to undertake the examination of this class of objects in the usual way requires a microscope with the addition of a polariscope, which calls for an outlay of at least fifty dollars, besides the cost of the objects; and while it is believed that such an outlay would be indirectly, if not directly, profitable, it is not necessary to expend a fiftieth of that amount to arrive at very satisfactory results.

The cost of the compact and efficient little instrument shown in Fig. 26 is as follows: 1 pocket magnifier, having two lenses 1½ in. and 2 in. focus respectively, giving when combined a ¾ in. focus, 50 cents; eighteen elliptical microscope cover glasses for analyzer, 38 cts.



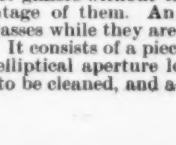
26

POLARISCOPE FOR MICROSCOPIC OBJECTS.

The cost of wood for the principal parts, the pasteboard tubes, the glass for the polarizer, and the metal strips for the slide-holding springs, can hardly be counted, and the labor must be charged to the account of recreation; so that less than one dollar pays for an instrument that will enable its owner to examine almost the entire range of microscopic polariscope objects with a degree of satisfaction little less than that afforded by the use of the best instruments.

The form, proportions, and material of the body of the instrument are entirely matters of individual taste. In the present case, the hand piece and sliding stage are made of ⅞ in. mahogany, the handle being formed on the hand piece by turning. The stage is 2¼ in. square, and has in its lower edge a half inch square, transverse groove, which receives the square rod projecting from the hand piece at right angles. The rod is held in the groove by a wooden strip fastened to the lower edge of the stage by two wood screws, so that it bears with a light friction on the under side of the rod.

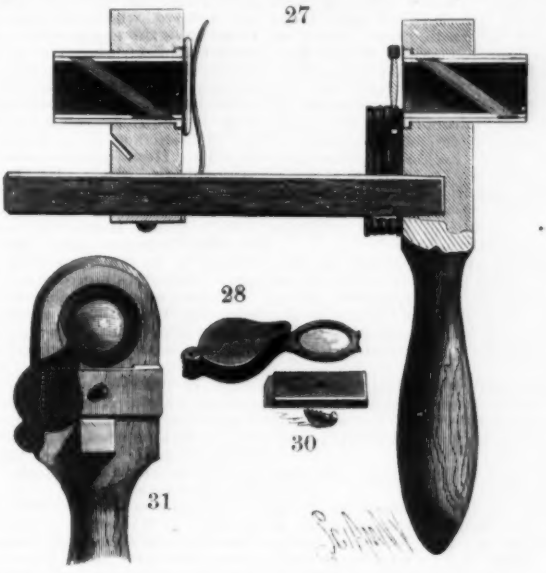
The hand piece and stage are both pierced above the rod with holes which are axially in line with each other. The diameter of the holes is governed by the size of the cover glasses. Those in the instrument shown are of the exact size and form of the annexed diagram.



These cover glasses are procurable from any dealer in supplies for microscopists. Eighteen of them, at least, are required. The paper tube inclosing these glasses is a little more than 1¼ in. internal diameter; its outside diameter is ⅞ in. and its length is 1¾ in. A narrow paper collar is glued around one end of the tube, and both the hand piece and the stage are counterbored to receive the collar, as shown in the sectional view, Fig. 27. To the tube thus described is fitted an internal paper tube, which is about ⅛ in. shorter than the outer tube. The inner tube is divided diagonally at an angle of 35° 25', which is the complement of the polarizing angle for glass (54° 35'). The oblique surfaces thus formed, when placed in the tube in opposition to each other, support between them the glass plates at the polarizing angle. The simplest way to arrange the angles of the tubes and other parts of the instrument is by the employment of a triangle of cardboard like that illustrated in Fig. 32. In fact, a copy of the triangle here shown may be used.

It is sometimes a matter of considerable difficulty to clean the thin cover glasses without the risk of breaking a large percentage of them. An effective device for holding the glasses while they are being cleaned is shown in Fig. 29. It consists of a piece of thin Bristol-board, having an elliptical aperture loosely fitting the edges of the glass to be cleaned, and a plain card glued to the back of the apertured card, and forming the bottom of the shallow recess into which the glasses are dropped for cleaning. The holder may be pressed down upon the table by the fingers of one hand, while the glass is rubbed with a soft linen handkerchief, after being breathed on. Glasses that cannot be easily and thoroughly cleaned in this way are worthless for this purpose.

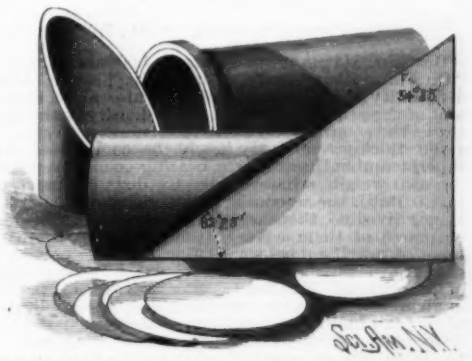
Before the glass plates are put together, they are dusted with a camel's hair brush to remove any adhering lint and dust. The paper tubes are made dead black inside and outside.



LONGITUDINAL SECTION OF POLARISCOPE AND DETAILS. HALF SIZE.

The front of the stage is provided with a pair of thin brass springs, which serve to clamp the object slide with a light pressure to the stage. In the back of the stage, below the central aperture, is formed a groove for receiving the black glass polarizing plate. The groove supports the black glass at an angle of 54° 35' with the plane of the stage, or at an angle of 35° 25' with the holes in the stage and hand piece. The polarizing plate may consist of a plate of polished black glass, but it is generally more convenient to employ an ordinary piece of glass blackened on one side. A thin pine wedge cemented to the back of the plate causes it to bind in the groove of the stage.

32



TRIANGLE AND PAPER TUBE. FULL SIZE.

To the inner face of the hand piece is clamped an ordinary pocket magnifier, shown in Fig. 28, by means of the wooden clip shown in Fig. 30. Fig. 31 shows the arrangement of the magnifier relative to the analyzer. Any magnifier of suitable focus may be pressed into the service. The face of the stage and other parts of the instrument visible through the analyzer are blackened.

29

HOLDER FOR GLASS.

The object to be viewed is placed on the stage and focused, when the instrument is held so that the black glass polarizing plate reflects the light through the object and through the analyzer. The analyzer is then turned, and the object observed. To heighten the color effects, a plate of selenite or mica may be placed immediately behind the object, or between the stage

and black glass plate. Mica plates of suitable thickness are selected by trial in the instrument, and preserved for future use.

It is sometimes desirable to rotate the polarizer. When the black glass plate is used, this is impracticable, but by removing this plate, and inserting in the stage a polarizer consisting of a tube containing plates like the analyzer, the effects of rotating the polarizer may be observed. To render the rotation of the paper tubes smooth and uniform, their bearings in the hand piece and stage are rubbed over with the point of a soft lead pencil, imparting to them a thin coating of plumbago, which diminishes friction and prevents sticking. The objects which may be examined by the aid of this instrument are very numerous. Many of them are easily prepared, and some need no preparation at all. The chemical salts mentioned below may be prepared for observation by allowing their solutions to evaporate on a slip of glass: Alum, bichromate of potash, bichloride of mercury, boracic acid, carbonate of potash, carbonate of soda, citric acid, chlorate of potash, hyposulphite of soda, iodide of potassium, nitrate of ammonia, nitrate of copper, nitrate of soda, oxalic acid, prussiate of potash (red), prussiate of potash (yellow), sugar, sulphate of copper, sulphate of iron, sulphate of nickel, sulphate of potash, sulphate of soda, sulphate of zinc, tartaric acid.

Slips of glass, 1 × 3 inches, are convenient for this purpose. A circle about ¾ inch diameter is formed on each slip with a piece of paraffin or wax, and while the slips are supported in a level position, a few drops of a rather strong solution are placed in each circle, and the slips are allowed to remain quietly until the crystals form.

For methods of covering and preserving these crystals, as well as for hints on the preparation of the more difficult crystals, the reader is referred to the works on microscopy, as these matters are without the province of this article.

The following vegetable and animal substances may be shown by polarized light:

Cuticles, hairs, scales from leaves, fibers of cotton and flax, starch grains, thin longitudinal sections of wood, oiled; spicules of sponges and gorgonia, cuttle fish bone, hairs, quills, horn, finger nail, and skin. These objects should be thin and translucent or transparent. It is necessary in some cases to increase their transparency by soaking them in oil or some other suitable liquid.

Many rock sections and sections of minerals may be studied advantageously by the aid of polarized light, but since the objects are quite difficult to prepare, no list of them is given.

It is perhaps well to suggest that the polarizer and analyzer shown in Fig. 27 may be readily adapted to a microscope, placing the polarizer below the stage and the analyzer in the draw tubes.

A LIMIT TO THE HEIGHT OF THE ATMOSPHERE.*

By HENRY A. MOTT, Ph.D., LL.D.

DR. WM. HYDE WOLLASTON, in his famous paper, "On Finite Extent of the Atmosphere,"† says: "If the divisibility of matter be infinite, so also must the extent of the atmosphere. For if the density be throughout as the compressing force, then must a stratum of given thickness at every height be compressed by a superincumbent atmosphere, bearing a constant ratio to its own weight, whatever be its distance from the earth. But if the air consist of any ultimate particles no longer divisible, then must expansion of the medium composed of them cease at that distance where the force of gravity downward upon a single particle is equal to the resistance arising from the repulsive force of the medium." And Dr. Lardner‡ says: "If a particle of air were raised above this height by the application of any external agency and then disengaged, it would drop by its gravity to the surface of the atmosphere, in the same manner and by the same law that makes a stone drop to the ground."

Prof. Mattieu Williams, in 1870, published a book bearing the title "The Fuel of the Sun," in which he attempts to account for the sun's heat, and he claims that the correctness of his position rests upon the question: "Whether the atmosphere which surrounds our earth is limited or unlimited in extent."

Williams claims that our atmosphere is but a portion of a universal medium which is distributed by gravitation. To obtain the weight of the atmosphere of any body in space, it is only necessary, according to Williams, to "multiply the mass of the body expressed in units of the earth's mass by its own square root, and the product is the total weight of the atmosphere of the body expressed in units of the earth's atmosphere, or $x = m\sqrt{m}$, where x is the atmosphere of the body expressed in units of the earth's atmosphere, and m is the mass of the body expressed in units of the earth's mass." One of the arguments against a universal atmospheric medium is the fact that the most delicate tests for a lunar atmosphere give no evidence that one exists.

If the moon had an atmosphere, just before the limb of the moon appears to reach a star § the latter would be seen through the moon's atmosphere, if there was one, and would be displaced in a direction from the moon's center. But the most careful observations have failed to show the slightest evidence of any such displacement. The spectra of stars when about to be occulted have also been examined, in order to see whether any absorption lines which might be produced by the lunar atmosphere became visible.

The evidence in this direction has also been negative.

If there is a lunar atmosphere, it is too rare to exert any sensible absorption upon the rays of light. It is also stated in some astronomical treatises that a lunar atmosphere having a density equal to only $\frac{1}{2000}$ of that of our own would be indicated by the acceleration of the observed period of an occultation. In answer to this Williams says: "This calculation rests upon the assumption of an atmosphere suddenly and sharply terminating in an absolute vacuum."

The lunar atmosphere is, as Williams maintains, only a graduated condensation of the general atmospheric medium, and he calculates that the pressure of the lunar atmosphere could only be $\frac{1}{70}$ of that upon the earth's surface; the barometer would stand six-tenths of an inch (15·24 mm.) instead of 30 inches (762 mm.), and the pressure $\frac{1}{70}$ lb. (136·066 grms.) instead of 15 pounds (6,803·29 grms.) per square inch, hence the lunar atmosphere would be equal to a vacuum in the receiver of an old fashioned air pump.

Williams claims that the gravital attraction of the earth would go on accumulating an atmosphere until such attraction was neutralized by that of some other orb, toward which an imaginary course was tending. If we cross the neutral line, and continue farther in the same direction, we enter the domain of another world, where the order of proceeding density is reversed. He calculates the pressure of the atmosphere of the sun at 32,903,928 lb. (14,923,906,601·68 grammes) per square foot, or about 230,000 tons (108,634,214·4 kilogrammes) on the surface of a human body; in atmospheres, 15,233·3. He calculates where the pressure of our atmosphere would be one grain (0·06479 grm.), the bulk of an equal weight will be 105,200 times greater than at the earth's surface. One grain (0·0679 grm.) there will occupy the space of 15 lb. (6803·39 grains) here. One pound (453·55 grammes) would be 15 times more rare, and one ounce (28·35 grms.) 240 times more rare.

Wollaston, after calculating the atmosphere of the sun, concludes that its extent would be so great as to visibly affect the apparent motions of Mercury and Venus when their declination makes its nearest approach to that of the sun. No such disturbance being actually observable, he concludes that an atmosphere cannot exist. He also calculated the atmosphere of Jupiter, and found it to be so great that its rarefaction would be sufficient "to render the fourth satellite visible to us when behind the center of the planet, and consequently to make it appear on both (on all) sides at the same time."

Williams shows that Wollaston's calculations are incorrect, as Wollaston's mistake is based on the assumption that the atmospheric pressure and density at any given distance from the center of the given orb will vary inversely with the square of that distance. As the area of the base upon which such pressure is exerted varies *directly* with the square of the *distance*, the total atmosphere above every imaginable starting distance would thus be ever the same. Williams also claims that the atmosphere in the interplanetary spaces will be subject to the same laws of motion as the solid planetary matter. Just what its density would be he has not deduced.

Observed and calculated movements of Encke's and Halley's comets seem to indicate the existence in planetary space of a medium—as an appreciable and measurable degree of mechanical resistance to motion of the attenuated matter of which they are composed is observable. This resistance may, however, at some future time be attributed to some other cause, which at present I will not stop to discuss.

Let us now consider the various heights which have been assigned to the earth's atmosphere by various investigators.

Sir John Robinson* says: "It is easy to show that the light which gives us what we call twilight must be reflected from the height of at least fifty miles, for we have it when the sun is depressed 18 degrees below the horizon." The limit concluded, however, is 45 miles (72·4 kilom.); but Robinson says: "A very sensible illumination is perceptible much further from the sun's place than this, perhaps twice as far, and the air is sufficiently dense for reflecting a sensible light at a height of nearly 200 miles (321·8 kilom.).

Laplace states that at a height of 52,986 meters, or less than 33 miles (53,097 meters), the atmosphere is thinned out to the utmost degree of rarefaction obtainable in an air pump.

M. Quetelet has inferred the existence of what he calls the "stable atmosphere," occupying the region extending from forty to eighty miles above the earth's surface.

Newcomb and Holden† state: "As shooting stars and meteors commonly commence to be visible at a height of about 160 kilometers, or 100 statute miles, the earth's atmosphere must rise to this height."

Lockyer‡ states: "There is evidence to show that we have an atmosphere of some kind at a height of 400 or 500 miles" (643·2 to 804·6 kilom.).

Prof. Alex. Buchan states§ that: "The limit of the atmosphere will be reached at where the force of gravity downward upon a single particle is equal to the resisting force arising from the repulsive force of the particles. From the observations of luminous meteors, it is inferred that the height of the atmosphere is at least 120 miles (193 kilom.), and that in an extremely attenuated form it may reach 200 miles" (321·8 kilom.).

Prof. David Trowbridge‖ claims that if the earth were 490° F. (272° C.) warmer than it now is, the height of the atmosphere would be nearly doubled; and he fixes the present height at 343 miles (551·887 kilom.).

Prof. Alf. Daniel,¶ speaking of our atmosphere, says: "At a height of 210 miles (337·89 kilom.), the single molecules are relatively so few (1,000 to the cubic cm., or 0·061016 to the cubic in.) that each molecule might travel through a uniform atmosphere of that density 60,000,000 miles [96,540,000 kilom.] without entering into collision; beyond a height of 300 miles [482·1 kilom.] the atmosphere is so rare (less than one molecule per cubic foot—28·32 cubic dec.) that the particles might freely travel through such an atmosphere from one fixed star to another; while in the fields of space, at distances practically infinite from the earth or from any other star, the number of cubic miles containing a single molecule would be represented by the figure 1 followed by 314 ciphers.

"This opens to us," says Daniel, "an extraordinary view of the nature of our atmosphere. We must, though the process cannot be rapid—for each particle rising from the earth is retarded by gravity, and falls back toward the earth—constantly be losing particles of nitrogen and oxygen as we are dragged through space, and we may constantly be picking up new ones. If we entered regions of space in which there were no particles fit to make up our losses, it would be an interesting question how short a time would suffice altogether to deprive us of our atmosphere."

Williams, speaking of the assumed limit to the height of our atmosphere, and the *supposed* luminiferous ether, says: "What must be the action of such a resisting medium [ether] upon the supposed boundary atoms* of the atmosphere of our planet, as it is rushing through it with its orbital velocity of nearly two millions of miles [3,218,000 kilom.] per day? Obviously, to brush them off the surface of the atmospheric ocean, and leave them deposited in the midst of the luminiferous ether, as a gale of wind lodges the sea spray on a lee shore. The outer atoms, thus removed, would leave the rest below in the same condition of unstable equilibrium; for according to this atomic theory, it matters not how great or how little be the extent of such an atmosphere of aggregated atoms—the outer layer must be subject to no pressure beyond that of its own gravitation, and that must be neutralized by its own elasticity and that of the next below. Thus would this newly exposed stratum of atoms be swept away by the merciless ether; then another and another again, till the planet would be stripped bare to the bottom of its dry land valleys. Then the ocean, relieved of the pressure which restrains the volatilizing power of the sun's rays, would spring upward into the condition of gaseous elasticity, forming another atmosphere subject to the same laws as the first, which would in like manner be swept away from the surface of the earth, leaving it all in a condition of arid, lunar barrenness."

Even if the ether were traveling in company with the earth in its orbit, there still remains the earth's axial motion; and then the whole solar system is traveling through space at the rate of 400,000 miles (643,600 kilom.) per day, all of which would tend to strip the earth of an atmosphere.

I will state right here, however, that while all this appears reasonable if such a medium as the ether existed, still I do not admit of the existence of any such medium, nor do I see the least necessity for my doing so. My reason for this is foreign to the subject we are discussing, consequently I will proceed to show by entirely new methods:

1. That *if there is* a definite height to the atmosphere, such height cannot exceed a given height to be mentioned.
2. That *there is a definite height* to the atmosphere.
3. What that height *probably* is.

It is well known that the earth, by its revolution on its axis, has on its surface at the equator an acceleration of 0·1112 foot (3·3908 cm.) per second directed toward the center of the earth, and therefore in the same direction as the acceleration of a body falling *in vacuo* at the equator, which is 32·2 feet (978·1028 cm.) at sea level. The absolute acceleration of the body is therefore the sum of these two, or 32·3112 feet (98·4036 cm.) per second, hence this would represent the *true* intensity of gravity at the equator, which would be demonstrated if the earth ceased to revolve around its axis.

The weight of all bodies would be increased (981·4036 + 3·3908) in the ratio of 288 to 289.

It therefore follows that if the earth revolved $\sqrt{289} = 17$ times faster than it does now, loose objects at the equator would fly off its surface.

To ascertain the height above the surface of the earth the atmosphere would have to be so that the upper layer would have a velocity 17 times greater than the velocity of the earth at the equator, is a very simple calculation, and we find that a globe having a radius of 67,384·3 miles (108,432·63 km.) would give the result required. This would give the height of the atmosphere, *in this case*, 63,420·88 miles (102,054·24 km.), the outer layer of which would have a velocity of (1,521·8476×17) 25,871·409 feet (7,885·6 meters) per second; but no such velocity or height is necessary, for the centrifugal force generated is 1·881 ft. (0·573 meter), which is too great for the attraction of gravitation at this outer layer, which is only 0·1114 ft. (3·395 cm.), while at the earth's equator it is 32·2 ft. (981·4036 cm.) per second; so that a body weighing one pound (453·55 grms.) on the earth's surface at the equator would only weigh 24·2 grains (1·568 grms.) at the height of 63,420·88 miles (102,054·24 km.).

As I have had occasion to use the term "centrifugal force," and will have to use it again, and as long as there has been considerable objection raised by Tait, Daniel, and others to the expression, it may be well for me, before proceeding, to explain what I mean by such an expression.

If a stone be whirled around like a slingstone by a string, the tangential velocity may be resolved into two components—one acting along the circle, and one away from it. The tension or stress in the string balances the centrifugal component of tangential velocity at every moment. If the string should snap, the stone flies off in a tangential direction, and not from the center, hence some physicists have been led to say that there was no such thing as "centrifugal force," as no such force is counteracted by the tension of the string.

Now, the fact is, the stone exerts a force upon the string which is necessary, or it would not deflect into a curve, but would persevere in rectilinear motion. This force exerted is, in my judgment, properly designated centrifugal force, as the derivation of the word indicates. It is not a force acting on the stone, but a force exerted by the stone on the string. It must be perfectly evident that if there had been no circular motion around a center (from which the word centrifugal is derived), there could have been no tangential effect on the stone when the string snapped; it therefore follows that scientists should not quibble on the application of this term.

Now to proceed: When $\frac{v^2}{r}$, that is to say, the centrifugal force due to the earth's rotation, is equal to g, gravity then is equal to 0 ($g = 0$). It is interesting to know, then, at what height above the earth's surface will $\frac{v^2}{r} = g$.

By the following formula, deduced by my associate, John Higgins, we arrive at the result.

Letting x = the number of times the earth's radius

---

* Read before the New York Academy of Sciences, November, 1886.

† Phil. Trans., 1822.

‡ Handbook of Nat. Phil.

§ Astronomy, Newcomb and Holden, p. 331.

* Ency. Brit., Pneumatics, 8th edition.

† Astronomy, p. 380.

‡ El. of Astronomy.

§ Ency. Brit., 9th edition.

‖ Of Waterburg, New York; see Record Sci. Ind., 1878, p. 161.

¶ Prin. of Phys., p. 204.

* Meaning molecules.

at the equator must be taken for the requisite height, we have:

$$\frac{32{\cdot}3112}{x^2} = \frac{v^2}{r}$$

$$\frac{v^2}{r} = \frac{\left(\frac{2\pi r}{86{,}400}\right)^2}{r} = \frac{\left(\frac{2\pi 3{,}963{\cdot}42\,x}{86{,}400}\right)^2}{3{,}963{\cdot}42\,x}$$

$$\frac{32{\cdot}3112}{x^2} = \left(\frac{\left(\frac{2\pi 3{,}963{\cdot}42\,x}{86{,}400}\right)^2}{3{,}963{\cdot}42\,x}\right) \times 5{,}280$$

from which we deduce:

$$x = 6{\cdot}634 \text{ nearly};$$

then, when

$$x = 6{\cdot}634,$$

we have

$$G = 0{\cdot}73418 \text{ feet per second.}$$
$$\text{C. F.} = 0{\cdot}73419 \quad " \quad "$$

Hence, if we imagine the atmosphere to extend 22,329·908 miles (35,928·82 km.) above the surface of the earth, then the velocity of the upper layer will be 10,085·06 feet (3,073·93 meters) per second.

But the centrifugal force $\frac{v^2}{r}$ at that height is 0·7341,* while the attraction of gravitation is also 0·7341.† Hence, at a height 6·634 times the radius of the earth from its center, $\frac{v^2}{r} = g$. Now for the application of this result: Since the centrifugal force is equal to or slightly greater than gravital attraction in the upper layer of *such an atmosphere*, it naturally follows, if matter is regarded as heterogeneous—composed of molecules and atoms—and not homogeneous, any block or layer of air above this height would be whirled off into space. Hence we have demonstrated that unless the atmosphere fills the whole of space, it could not extend above the earth to a height exceeding 26,293·328 miles (42,305·96 km.) above the center of the earth, or 22,329·908 miles (35,928·82 km.) above the surface of the earth at the equator: for if any attempt were made to so extend it, each layer or block of atmosphere added would, as just stated, be whirled off into space, thus leaving the height of this *supposed* atmosphere always less than 22,329·328 miles (35,928·82 km.) in height. The weight of a mass of matter weighing one pound (453·50 grms.) at the earth's surface would be at this height 159,065 grains (10,396 gms.).

I will now proceed to show that there is a limit to the height of the atmosphere, and thus comply with the second proposition.

Williams‡ claims that the elasticity of the atmosphere is unlimited; and Daniel§ says: "Thus a gas, with its *tendency to indefinite expansion*, has elasticity of volume."

Now it becomes necessary, in studying the true height of the atmosphere, to investigate the question of *unlimited elasticity* and *indefinite expansion*, and this leads us to the consideration of bodies deprived of all heat, or at the absolute zero of temperature. In this condition, if the body was free from porosity, we should find the most compact condition that matter could exist in. Just how many heat units would have to be taken from a body to reduce it to absolute zero is not known as a fact; it is, however, assumed that as gas expands $\frac{1}{273}$ of its volume for every Centigrade degree (or $\frac{1}{491{\cdot}4}$ for every F. degree) of added heat, therefore −273° C. (−459 4° F.) would represent the absolute zero, so that at −274° C. a body would be totally deprived of heat; whether this temperature is correct or not we do not know. The *normal* condition, then, of *all* bodies is the solid deprived of heat, and this was first pointed out by Dr. A. Wilford Hall.* A liquid or a gas is an abnormal condition of matter, and they owe their existence as such to the heat within them. If sufficient heat were to be taken from a gas, it would become a liquid; and if taken from a liquid, it becomes a solid; and if all heat be taken from the solid, the *normal* contracted condition of matter is arrived at.

Heat, therefore, expands matter, and changes its form and properties. Before proceeding, it will be necessary for me to state that I consider matter perfectly homogeneous throughout, containing, in most cases, porosity, but being free from *supposed* molecules and atoms, and I claim that experiment and reason dictate that matter is theoretically infinitely divisible.

I also claim that the forces of nature are entities, real, objective things, although not material. We give to matter (for one reason) objective resistance, because it cannot be destroyed—its quantity cannot be altered. We must be just as consistent with respect to *force*, which is likewise indestructible, and the quantity of which cannot be altered. I define *energy* as the power or ability of doing work possessed by force.

Force is the entity, not energy. Without further elaborating these points, I will proceed. Heat expands matter, but the expansion is not the result of separating portions of it from other portions, but every portion, however small, down to infinity, is expanded, just as a rubber balloon is expanded by blowing air into it, and it will remain *definitely* expanded so long as a *definite* temperature is maintained. Just, then, as a piece of iron occupies a given definite volume at a fixed temperature, so will a gas occupy a given definite volume at a fixed temperature—*other* forces not acting on it—no matter if such a statement is contrary to modern theories. I am perfectly well aware of the fact that the molecular theory *assumes* there is no cohesive force present in a gas which is allowed freely to expand, or in a gas unless subjected to considerable pressure.

In my judgment, there is not a particle of truth in such a statement. When water at 212° F. (100° C.) is converted into steam of 212° F. (100° C.), it expands 1,695 times its own volume. In each volume of steam, in my judgment, there is just $\frac{1}{1695}$ as much cohesive force at work as there was in the water; and to prove this we have only to withdraw the heat, when the cohesive force *contracts* the steam sufficiently to convert it back into water containing as much cohesive force as it did at the start. To proceed: If a vessel be filled with oxygen containing sufficient heat to enable it to exist as a gas (for less heat would permit it to exist as a liquid or solid), and the temperature of the gas, the vessel, and the surrounding air be found to be 21·11° C. (70° F.), the oxygen at this temperature will exert a given fixed pressure on the sides of the containing vessel, owing to the heat present in it, which has expanded (stretched it, so to speak) to that extent; it is the expansive force due to heat that produces the pressure. To prove this, we have only to add more heat to produce greater pressure, or abstract heat to produce less pressure.

The idea that a gas possesses the property of infinite expansion without the exercise of an infinite amount of heat is contrary to all that experimental science has deduced or reason could dictate. After solid oxygen has received sufficient heat to permit it to be a gas at 21·11° C. (70° F.), it can expand no more than to a definite fixed volume, unless acted upon by some force which could overcome the forces of cohesion and gravity, which tend to prevent any increase in bulk. It must not be assumed that, because such a gas when liberated in the atmosphere goes on distributing itself throughout the air, such action is evidence that oxygen has no fixed volume at 21·11° C. (70° F.). It might just as well be said of a piece of India rubber four inches in length that such a piece has no specific form at 21·11° C. (70° F.), because, if a force is applied at both ends in opposite directions, it can lengthen to twelve inches.

The fact is that the minute the oxygen under consideration, or, in fact, any gas, is liberated in the air, such gas is immediately acted upon by the attraction of adhesion, which tends to make it distribute itself as sugar is dissolved in water. But the extent of its distribution would be limited to the bulk of the air, as its volume is limited, and is kept so by the attraction of cohesion and gravity.

This brings us to the consideration of elasticity, and to disabuse our minds of the idea that elasticity is a force, and, therefore, can do something.

Elasticity, so far from being a force, is but an accidental property of a body, which within certain limits permits the same to expand after being compressed, or *vice versa*.

Bodies at absolute zero of temperature can have no elasticity. No non-porous body could be compressed at absolute zero, nor can it be compressed until it is expanded by heat; there can, therefore, be no elasticity until a body is expanded.

Elasticity is, therefore, a property matter possesses in virtue of its heat and cohesive force. Solids, liquids, and gases may be condensed in volume by the withdrawal of heat, cohesive force acting as the contracting power, or they may be reduced in volume by the expenditure of mechanical energy; in all such cases, the body will become heated, owing to the *crowding of the heat into a smaller volume*.

As we might expect, the extent of elasticity in solids is less than in liquids, and still less than in gases.

Perfection of elasticity and the *quantity or extent* of elasticity must not be confounded.

Dr. Hall* gives as a definition: "The *amount* of elasticity in any given body consists in the extent to which that body can be compressed with a given force, while possessing still the innate property of restoration to its original form when released from pressure."

The *quickness of recovery* is principally dependent upon the mechanical energy stored up as force, and also upon the *quality* of the elastic property of the body in permitting the reaction among its infinitely small particles by this stored up force, and not at all upon its *quantity or amount*.

When mechanical energy is used to compress a body, such energy is either stored up in the body or is converted into some other form of energy. In a body within the limits of its elasticity, the mechanical energy applied is stored up; when this limit is passed, the mechanical energy is converted into some other form of energy in the body, probably entirely into heat.

It is evident, then, that a gas is not possessed of *unlimited elasticity*, but is possessed of a limited amount of elasticity, dependent upon the extent to which the cohesive force has been overcome by other forces. I say other forces, for besides heat the force of adhesion in mixtures may have some indirect influence. Now let us apply this information to the problem before us.

The atmosphere is acted on principally by four forces, the attraction of gravitation, of cohesion, of adhesion, and by heat. The heat in the atmosphere tends to expand the air, and the temperature becomes reduced as the heat is made to do the mechanical work of expansion, which the attractive forces of cohesion, adhesion, and gravitation tend to resist; and when the heat remaining in the air can no longer overcome the action of these attractive forces, a limit is reached to the expansion of the air, *and the true height of the atmosphere is obtained*. It must not be forgotten, however, that the atmosphere travels around with the earth, and that the centrifugal component of tangential velocity tries to overcome the tension of gravity, and when it accomplishes this, any air which by some reason had lodged at such a height would, as has been already shown, be whirled off into space.

So, then, as I have proved the height of the atmosphere to be limited—as its volume depends upon the amount of heat in it—such height must be less than 22,329·908 miles (35,928·82 km.), as I have also demonstrated.

Now let us endeavor to deduce the height of the atmosphere, and thus comply with the third and last proposition. We cannot expect to get at the height exactly, but we certainly can approximate it very closely. If the earth was devoid of an atmosphere, we should be involved in total darkness from the instant of sunset till the instant of sunrise.

The transition, however, from day to night and from night to day occupies an interval which varies with the latitude and the declination of the sun. This intermediate stage is called twilight. Morning twilight is called *aurora*, and evening twilight *crepusculum*; the cause of both is the same. Twilight is due to the reflection and refraction of the sun's light by that portion of the atmosphere which is still illuminated by the sun's rays after he has set to us, or a similar portion on which he shines before he rises in the morning. At the equator twilight lasts 1 hour and 12 minutes, or $\frac{1}{20}$ of semicircumference, which is equal to 18°. When the sun has descended so far as not to illuminate any of the atmosphere which lies above the horizon, twilight is at an end. "It* is assumed when the sun has reached this position, in which no portion of air that lies above the horizon is directly illuminated, faint stars will become visible over the western horizon, and thus that the end of evening twilight is definitely marked by the appearance of such stars. In like manner morning twilight is astronomically defined as beginning when faint stars situated in the eastern horizon begin to disappear." Therefore as long as the sun is not more than 18° below the horizon, its light is reflected by the air, the clouds and vapors suspended in it, in sufficient quantity to render even distant objects visible.

If the earth's atmosphere were more extensive than it is, the twilight would of course be longer, since the sun would not cease to illuminate the higher portions of the atmosphere until more than 18° below the horizon; and if the atmosphere were less extensive, the reverse of this would be the case.

Knowing, then, the depression of the sun (18° mean) requisite for the cessation of twilight, the approximate extent or height of the atmosphere has been calculated. In the calculation, the curvature of the incident and reflected ray due to refraction must be taken into account. Without this correction, the result would be 49·3 miles; with it, however, about 45 miles in height.†

Sir J. Herschel‡ states that "a secondary twilight may be traced even beyond the height mentioned, consequent on a re-reflection of rays dispersed through the atmosphere in the primary one. The phenomenon seen in the clear atmosphere of the Nubian desert, described by travelers as the 'after glow,' would [he says] seem to rise from this cause."

We must bear in mind one fact, and that is that the atmosphere must have a certain density before light will affect it so as to send a sensible amount of reflected light to the eye of the observer through the intervening body of air.

The following table shows the density of the atmosphere above the earth at various heights:§

Heights above the sea in miles.	Height of the Barometer in inches.	Density of the air, sea level = 1.	Pressure in pounds to the sq. inch.
0	30	1	15
3·4	15	$\frac{1}{2}$	7·5
6·8	7·5	$\frac{1}{4}$	3·75
10·2	3·75	$\frac{1}{8}$	1·875
13·6	1·87	$\frac{1}{16}$	0·9375
44·2	0·0036	$\frac{1}{8192}$	0·0018
47·6	0·0018	$\frac{1}{16384}$	0·0009
51·	0·0009	$\frac{1}{32768}$	0·0004

At the height§ of 13·6 miles, the air would be rarer than hydrogen. At the height of fifty miles, the mercury would be elevated about one thousandth of an inch, and the air would be less than one-thirty thousandth of its density at the sea level, and, according to Prof. Norton, at this height the limit of the atmosphere is practically reached.

Prof. Norton says that "the intense cold of the upper limits of the atmosphere tends to diminish the expansion of the air, by diminishing the repulsion between its molecules, so that it is probable that the height does not exceed forty-five miles;" and he claims that this result is confirmed by the phenomena of refraction of the heavenly bodies. The speculation as to the distance a supposed molecule could travel without encountering another molecule at a height of 200 or 300 miles (321·8 to 482·7 km.), as given by Daniel, is shown by Prof. Norton to be erroneous, as the intense cold of the upper limits of the atmosphere would tend to diminish the expansion of the air by diminishing the repulsion between the molecules. Daniel does not seem to have taken this factor into account. As I have stated before, the minute that cohesive force and gravity get the best of the heat present in the air, that minute we arrive at the limit to the expansion of the air; and if we knew this point, as I have deduced the limit where the centrifugal force caused by the earth's rotation gets the best of gravity, we could easily assign to the atmosphere its true average height.

I say average, for the upper surface of the atmosphere must be like the surface of the ocean, always in wave motion, as the heat in the atmosphere fluctuates up and down.

It is a simple matter of calculation to show that if the atmosphere were everywhere of the same density as at the level of the sea, its height would be 5·2 miles; for the height of the atmosphere required to balance the column of mercury in the barometer would be 11,000 × 29·922 inches (742 mm.), or 27,400 feet (8·352 km.), where 11,000 represents the density of mercury compared with air.

The pressure of air may therefore be reckoned as equal to a column 5·2 miles (8·369 km.) high, having throughout a density equal to that of the air at sea level.

I have calculated the weight of the atmosphere, and find it to be 5,366,973,781,969,539·2936 tons, and have also calculated what its height should be if it had a uniform density, as that at sea level, and I have done this by an entirely new method.

Experiment has shown that 1·604 ounces (45·467 grms.) of solid oxygen occupies 2·82 cu. in. (46·25 c. c.), therefore one ton will occupy 32,587 cu. ft. (688·31 cu. dec.). It has also been found that 687·3 vols of gaseous oxygen are condensed into one in the solid form.

From this data, with a little labor, I have found that if the atmosphere consisted alone of oxygen gas, the height it would have at 32° F. (0° C.) and at density of sea level would be 4·53 miles (7·29 km.), instead of 5 miles (8·05 km.), as deduced above. As, however, four-fifths of the air is nitrogen, and a somewhat larger volume of nitrogen must be compressed into one to form the solid, the difference—0·67 of a mile—would be made up to 5·2 miles, which proves the correctness of the two calculations.

The next step is to find out from this data the height of the atmosphere in its expanded condition. This I have attempted to do also by a new method, which I will proceed to describe.

Taking the average temperature of the air as 60° F. (15·55° C.), and the temperature of space as −60° F. (−51·11° C.), as calculated by Fourier, we can call the average temperature throughout the atmosphere from the earth to its height one-half, or −33·33° C. (−27·98°

* Prin. of Phys., p. 224.
† See Record Sci. Ind., 1878, p. 161.
‡ Science in Short Chap., p. 75, Standard Lib.
§ Prin. of Phys.

* Dr. A. Wilford Hall.

* Norton's Astronomy, p. 156.
† See Johnson's Cyc., Twilight; S. Alexander.
‡ Outlines of Astronomy.
§ See Nat. Phil., S. A. Norton, p. 137.

F.). There will therefore be 33·33 Centigrade heat units to disappear as mechanical work doing expansion.

Now the next step is to find out the true value of a heat unit when all of its energy is used in expanding the air, and none of it in raising the temperature. We know that a gas expands $\frac{1}{273}$ of its volume for each rise in temperature of 1° C. (or $\frac{1}{491}$ for each rise in temperature of 1° F.); but in this case the heat unit not only has to do the work of overcoming cohesive force, and thus expanding the air, but of raising the temperature at the same time This result is not, therefore, what is required.

We can, however, from analogy deduce its value. When water at 100° C. (212° F.) is converted into steam of 100° C. (212° F.), its increase of volume, as stated before, is 1,695. This expansion is accomplished by 535·5 heat units, and from this we deduce the expansive value of a heat unit as equal to 3·12. Now, as we have 33·33 heat units, their total value would be 104. Applying this deduction to the number of cubic miles of atmosphere of equal density, which is 949,882,378·683 cubic miles, I have deduced the height of the atmosphere to be 49·83 miles, or, in round numbers, 50 miles (80·45 km.). This result not only agrees with the deductions made by Prof. Norton, but agrees with the deductions made from the phenomena of twilight, which fixes the height at 45 miles (72·4 km.). There is undoubtedly five miles of atmosphere of too small a density to reflect light, and this added to the 45 miles makes 50 miles (80·45 km.) as the probable height of the atmosphere.

As I have cited cases where astronomers have stated that meteorites have been observed at the height of over 100 miles (160·9 km.), it may be well before closing this paper to allude briefly to such results.

It must be admitted at the start that great difficulty arises when we try to calculate the height of meteorites, owing to their limited duration and the fact that they seldom leave a persistent streak of light; again, it is necessary that two observers must not only see the same meteor from different stations, but at the same time. All these considerations cast some distrust on the various heights to the atmosphere which have been assigned from such phenomena. It is perfectly evident that no meteorites have been seen as high as 210 miles, when we take the molecular theory of the constitution of matter into consideration; for Daniel* tells us that at such a height there are only 1,000 molecules in a cubic centimeter, and that each molecule can travel through a uniform atmosphere of that density sixty million miles without entering into collision. And Daniel further tells us that the supposed ether, according to Clerk Maxwell, has the same density as the air at that height, or $\frac{1}{1000000000000000}$ that of water. This being the case, a meteorite would not have to strike the atmosphere to be rendered incandescent, for its travel through the ether would accomplish this result.

It is not even reasonable to assume that a medium of one hundred-millionth of an atmosphere, or the density the air would have at about 90 miles (144·81 km.) above the earth, could offer sufficient frictional resistance to render a meteorite incandescent; for by calculation, it is easy to show that not even one per cent. of the space is occupied by the oxygen and nitrogen molecules, in fact only sixteen ten-billionths of the space is so occupied; the rest must be empty.

Even at the height of 68 miles (109·42 km.), where the density would be one millionth, it is not reasonable to assume sufficient frictional resistance, for the molecules at this height only occupy one hundred and sixty-one hundred-millionths of the space; that is to say, in a cubic foot of such air the oxygen and nitrogen would only occupy one hundred and sixty-one hundred-millionths of the space; the rest would be vacant so far as the constituents of the air are concerned.

At fifty miles (80·45 km.) in height we find that five hundred and seventy-three ten-millionths of the space† is occupied by the constituents of the air, and the density is about $\frac{573}{10000000}$ of that at the sea level. I do not think it is reasonable to assume that a medium less dense would be able to offer sufficient frictional resistance to a meteorite passing through it so as to heat the same to incandescence. For these reasons, and others that I have mentioned, I do not consider the various heights assigned to meteors by astronomers as reliable.

It must be understood that inasmuch as I do not believe in the molecular theory of the constitution of matter, but do believe that matter is perfectly homogeneous throughout, it naturally follows on this theory that at a height of fifty miles (80·45 km.) the air is still homogeneous, occupying the whole of the space; it being simply expanded, dilated, or thinned out, every portion being expanded just to the extent that the heat present has overcome the cohesive and gravital attractive forces.

To close this paper, I would state that I have established the fact *that if the atmosphere is limited* in height, the height must be below 22,329·1156 miles (35,928·82 km.); also that *the height of the atmosphere is limited*, and *that its probable height is fifty miles above the earth.*

DETERMINATION OF NITROGEN IN COAL AND COKE.

A NEW method has recently been arranged by Herr S. Schmitz, who applies the plan proposed by Kjeldahl. A quantity somewhat less than 1 gramme of the finely powdered coal (a smaller quantity still will do if coke is being experimented on) is placed in a ½-liter flask made of good potash glass, together with 1 gramme of finely powdered mercury oxide and 20 c. c. concentrated sulphuric acid, and heated to boiling point for three hours, or until dissolved. The cooled contents of the flask are then transferred to a ¾-liter flask containing a little water. To this is added from 120 to 140 c. c. of pure sodium hydrate solution, 35 c. c. of solution of yellow sodium sulphide, and a small piece of zinc. The flask is then connected by means of a bulb tube (to prevent spurting over) to an ordinary tube condenser, the other end of which dips into an excess of 1-20th normal sulphuric acid, contained in a small flask. A slight difference is made in the treatment of coke. After boiling for an hour with the mercury oxide and sulphuric acid, another gramme of the former is added, and 2 grammes of finely powdered potassium permanganate. The boiling is then continued as above. The additional quantity of mercury necessitates double the quantity of sodium sulphide solution in the distilling flask. The distillation is carried on for 20 or 30 minutes; and the sulphuric acid solution, of which an excess has been taken in the first place, is titrated with 1-20th normal barium hydrate and rosolic acid. Coke prepared in a platinum crucible may be treated as coal. Good results are obtained with coke that has been 24 hours in the coking oven; but if the coke has been in the oven 48 hours or more, the results are not reliable.

* Prim. of Phys., pp. 224 and 208.

† The number of molecules in 1 cu. c., according to Sir Wm. Thomson, is 6,000,000,000,000,000,000,000. In air of $\frac{1}{1000}$ density at 50 miles there would be 214,300,000,000,000,000. Molecules in cu. inch = 3,506,519,800,000,000,000. Diam. of molecule = $\frac{1}{1000000000}$ mm. Volume of a molecule = 0·000000000000000000028 cu. mm., 0·0000573 cu. inch, is the space occupied by all the molecules = $\frac{1}{17444}$ cu. in.

THE LANGHAUS BATTERY.

THE Langhaus battery is designed for utilizing carbon for the production of an electric current. In the Jablochkoff and Becquerel couples, in which carbon is employed for the purpose, a large quantity of the heat set free by the reaction is lost. The same is the case with the process that consists in separating metals from their oxides by carbon, or from their sulphides by means of some other metal.

Mr. Langhaus proposes to use this waste heat in the production of an electric current. The apparatus devised by him for this purpose is shown in the annexed figures. An earthen vessel, *a*, contains a

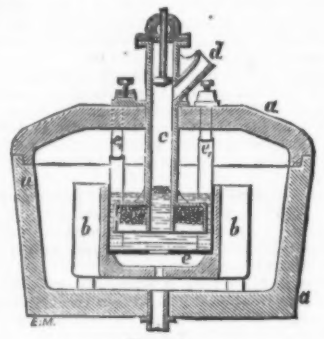


FIG. 1.

crucible of marble or carbonate of magnesia divided into two compartments by a partition that contains an aperture. The compartment, *b*, contains the oxides or sulphides, which, when melted, pass into the other compartment. Over this latter there is fixed a metallic cylinder, C, which is provided with a piston, and which widens out into a capsule below. This cylinder, again, carries a lateral tube, *d*.

Finally, under the widened portion of C, there is a metallic disk, *c*, connected with two supports, e_1, e_1, provided with terminals on the cover of the apparatus. The apparatus having been filled through the funnel, *f*, with the mass of oxides, and the latter having been

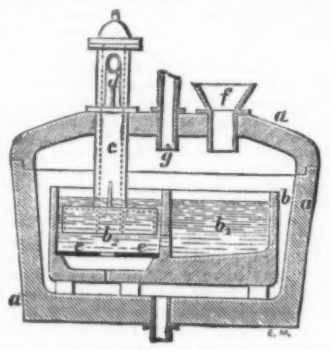


FIG. 2.

melted by the burner, *g*, the carbon is introduced through the tube, *d*. The carbon is then compressed by the piston, and, being lighter than the molten mass, enters the wide portion of the cylinder, C. A violent reaction then occurs, leading to the reduction of the oxidized mass.

The great quantity of heat liberated under these circumstances is capable of producing an electric current. The communications being established as shown in Fig. 3, we have a complete circuit formed of the cylinder, C, of the carbon, of the molten mass, of the disk, *c*, of the supports, e_1, e_1, and of the external circuit.

We have here, then, the proper conditions for the production of an electric current, as two different

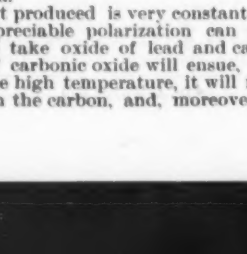


FIG. 3.

bodies, carbon and metal, are in contact with a decomposable liquid.

The current produced is very constant, since, in this case, no appreciable polarization can occur. If, for example, we take oxide of lead and carbon, a disengagement of carbonic oxide will ensue, but in consequence of the high temperature, it will not be able to fix itself upon the carbon, and, moreover, the deposit of metallic lead upon the disk will not cause any polarization. By means of the piston it is possible to renew the carbon without interfering with the operation of the apparatus.

The advantage of this proess is evident, since, in addition to the reduction of the metallic oxides or sulphides, it permits of utilizing under the form of electricity the excess of heat necessary to the reduction.—*La Lumiere Electrique.*

EFFECT OF ELECTRICITY ON FINE BALANCES.

M. HENNIG DE WURTZBOURG having noticed some incomprehensible differences in the weights of equivalent quantities, undertook some minute investigations, which showed that balances of precision are often influenced by the electric state of the glass cage which surrounds them. This electricity influences the small auxiliary slides which are placed upon the beam of the balance in order to make it more sensitive. The error resulting from this influence may amount to 600 milligrammes when the cage is strongly charged, and two hours afterward there may still be an error of ten milligrammes.

A CATALOGUE containing brief notices of many important scientific papers heretofore published in the SUPPLEMENT, may be had gratis at this office.

THE
Scientific American Supplement.
PUBLISHED WEEKLY.
Terms of Subscription, $5 a year.
Sent by mail, postage prepaid, to subscribers in any part of the United States or Canada. Six dollars a year, sent, prepaid, to any foreign country.

All the back numbers of THE SUPPLEMENT, from the commencement, January 1, 1876, can be had. Price, 10 cents each.

All the back volumes of THE SUPPLEMENT can likewise be supplied. Two volumes are issued yearly. Price of each volume, $2.50 stitched in paper, or $3.50 bound in stiff covers.

COMBINED RATES.—One copy of SCIENTIFIC AMERICAN and one copy of SCIENTIFIC AMERICAN SUPPLEMENT, one year, postpaid, $7.00.

A liberal discount to booksellers, news agents, and canvassers.

MUNN & CO., Publishers,
361 Broadway, New York, N. Y.

TABLE OF CONTENTS.

PAGE

I. CHEMISTRY.—Determination of Nitrogen in Coal and Coke.—A detailed account of Kjeldahl's method as applied to these substances by C. Schmitz.—A titration process for determination of nitrogen in the form of ammonia 9078

II. ELECTRICITY.—Effect of Electricity on Fine Balances.—An interesting note of the effect upon the accuracy of balances of the electrification of the glass of their cases.—A previously unsuspected source of error 9078
The Langhaus Battery.—A battery dependent on the action of a melted oxide or sulphide, the electrodes or plates being formed of carbon and metal.—A non-polarizing battery that only generates electricity on the application of heat.—3 illustrations 9078

III. ENGINEERING.—Voussoir Calculation and Measurement.—By C. POWELL KARR, C.E., Consulting Arch'l Architect, New York.—An elaborate article, treating of the methods of determining the factors of stone arches.—Empirical formulæ, with examples of actual bridges.—Full illustrations of graphic methods of laying out work.—Summary of methods and necessity for exactness of the work of calculation.—14 illustrations 9066

IV. MINING AND METALLURGY.—Massick & Crookes' Hot Blast Fire Brick Stove.—A recent hot blast apparatus as erected at the Ashkam-in-Furness Iron Works.—Simplicity of its construction, permitting the use of ordinary fire brick.—Thorough absorption of the heat of the waste gases.—Number of these stoves now working.—4 illustrations 9070
The "Morgan" Miner's Safety Lamp.—An improved lamp of comparatively high illuminating power.—The recipient of a gold medal at the International Inventions Exhibition.—The question of its efficiency and its claims to success.—2 illustrations 9071

V. NAVAL ENGINEERING.—Launch of the French Ironclad Le Hoche.—Detailed account of this launch; the question of support; friction on the ways; the absence of lateral inclination; success of the operation under the supervision of M. Huin, the designer 9065
Triple Expansion Marine Engines.—A paper read by Mr. ROBERT WYLLIE, of Hartlepool, at the Leeds meeting of the Institution of Mechanical Engineers.—Full discussion of the subject, including among other heads the consideration of steam velocities, piston valves, sequence and number of cranks, artificial draught, and practical results attained on steamers 9064

VI. ORDNANCE.—Recent Improvements in the Manufacture of Rifle Barrels.—Abstract of a paper read at the last meeting of the British Association by Mr. ARTHUR GREENWOOD, M.I.C.E.—System of manufacture adopted at the Enfield factory; simultaneous drilling of three barrels from both ends; form of the drill bits; speed of rotation of the barrels; other details of the process 9070

VII. PHYSICS.—A Limit to the Height of the Atmosphere.—By HENRY A. MOTT, Ph.D., LL.D.—A long discussion of this question; the limit of height of the atmosphere determined by the action of centrifugal force upon its outer layers; full mathematical exposition 9076
Simple Experiments in Polarized Light.—By GEORGE M. HOPKINS.—An exhaustive treatment of the experimental study of this subject; observation of the effects of polarization with apparatus of the simplest as well as of the more complex kind; home experimentation, and new objects for illustration of its laws; the most abstruse branch of the physics of light simplified; illustrations of polarized crystal sections.—26 illustrations 9072

PATENTS.

In connection with the **Scientific American,** Messrs. MUNN & Co. are solicitors of American and Foreign Patents, have had 42 years' experience, and now have the largest establishment in the world. Patents are obtained on the best terms.

A special notice is made in the **Scientific American** of all inventions patented through this Agency, with the name and residence of the Patentee. By the immense circulation thus given, public attention is directed to the merits of the new patent, and sales or introduction often easily effected.

Any person who has made a new discovery or invention can ascertain, free of charge, whether a patent can probably be obtained, by writing to MUNN & CO.

We also send free our Hand Book about the Patent Laws, Patents Caveats, Trade Marks, their costs, and how procured. Address

Munn & Co., 361 Broadway, New York.

Branch Office, 622 and 624 F St., Washington, D. C.